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Research Article

Computer Aided Pressure Transient Analysis of a Layered Reservoir System with a Constant Pressure Boundary

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Abstract:

A Layered reservoir system with a constant pressure bottom boundary has immense potential to produce oil and gas, using horizontal wells may further increase productivity because it offers a larger surface area for fluid withdrawal. Test analysis in this system is tedious, time consuming and interpretation may be confusing or erroneous. Hence an efficient algorithm is needed to generate accurate pressure response of the wells to facilitate quick and easy analysis and prediction. This paper presents an interactive computer program that computes the dimensionless pressure and dimensionless pressure derivatives of horizontal wells in a two layered reservoir system with a constant pressure boundary. Program's codes and structure of the reservoir model solution were written utilising the exponential integral solution, Gauss Laguerre and Gauss Lagrange numerical methods. The program has features that gave visual interpretation (graphical user interfaces (GUI)) as it computes pressure and derivative values when the interface between the layers is either permeable (crossflow reservoir) or impermeable (no crossflow reservoir). Examples illustrating its use in model identification and well test analysis of layered system with constant pressure boundary are also included in this work. Hence it will serve as a good learning tool because it is user friendly, fast and results have acceptable level of accuracy.

1. Introduction

The petroleum industry has witnessed a massive increase in software applications in recent times, though computers cannot replace human judgement, they have become indispensable to the petroleum engineer in the area of oil and gas prediction, understanding complex processes, planning maintenance and forecasting. Most software are problem or process specific (customised to solve unique challenges) while others are more general and available for public use [1, 2 ,3]. The use of computer application in pressure transient analysis has numerous advantages over manual analysis. The benefits include increased productivity and improved quality of test and analysis in the shortest possible time [4, 5, 6 and 7]. Layered reservoirs analysis are complicated, due to the existence of different layers with varying permeability, porosity and formation characteristics, pressure transient test analysis of these systems are difficult. Different mathematical methods employed to obtain solution to the flow equations for the layered models, include the Laplace transform, finite difference, modified Bessel function, linear regression analysis and the source function method [8, 9,10 and 11]. For these systems with complex configuration, robust computer programs are needed to ease analysis and interpretation. In this work the mathematical solutions to the reservoir models were presented. The models were developed using the Greens and Source function method to model the two layered reservoir subject to a bottom waterdrive using horizontal wells. An efficient and fast computer algorithm was developed and presented. This algorithm can compute fast and accurate values of pressure and pressure derivative of a two layered reservoir system. In addition a visual display of the solution for quick and easy interpretation of the results was provided. The program created a platform where different configurations of the reservoir and horizontal wells can be envisaged and evaluated. It has the capacity to compute the number of flow periods that will prevail for a particular set of well/reservoir parameters, crossflow coefficient, modification factor and total system permeability. Graphical representations of results were also provided. Typical numerical examples were used to test the performance of the program and highlight its application in well test analysis of layered system with constant pressure boundary

2. Design of Software Model

The two layered reservoir model with horizontal wells developed in [12] was used in this work. Relevant equations such as equations (9)–(19) were employed in the design of the software using programming technique of Visual Basic.Net [13].

2.1 Dimensionless parameters

For simplicity and easy comparison with other works, parameters were converted to their dimensionless form. Equation (1) was adopted to model dimensionless distances of the reservoir and the well. Effective dimensionless wellbore radius was modelled using equation (2) [14]

$$i_D = \frac{2i}{L}\sqrt{\frac{k}{k_i}} \qquad (1)$$

$$\mathrm{R}_{eD} = \frac{r_w}{L}\left(\sqrt{\frac{k}{k_z}} + \sqrt{\frac{k}{k_y}}\right) \qquad (2)$$

Dimensionless time and pressure and pressure drop were modelled using equation (3) ,(4) and (5) respectively [15]

$$\mathrm{t}_{\mathrm{Di}} = \frac{\mathrm{Kt}}{\varphi\mu \mathrm{C}_{\mathrm{t}}\left(\frac{L}{2}\right)^2} \qquad (3)$$

$$p_D = \frac{2\pi kh\Delta p}{q\mu} \qquad (4)$$

The pressure drop caused by production from a continuous source in a well is expressed as

$$\Delta p(x,y,z,t) = \frac{1}{\phi C_t}\int_0^t q_L.S(x,y,z,t)dt \qquad (5)$$

Where s(x, y, z, t) represents the instantaneous source function (ISF) for the particular reservoir and well configuration.

2.2 Layered system equations

Source and Greens Function were selected to assemble each layer's dimensionless pressure(P_{Di}) using the Newman's product rule]. Utilising the superposition principle the full P_D for the well and reservoir system was evaluated as (6) and (7). Derivatives of the P_D were computed using equation (8).

$$P_{D1}\left(X_D,Y_D,Z_D,\tau\right) = 2\pi h_D\int_0^{t_D} S\left(X_{D1},\tau\right).S\left(Y_{D1},\tau\right).S\left(Z_{D1},\tau\right)d\tau_D \qquad (6)$$

$$P_{Di} = P_a + P_b + ...P_n \qquad (7)$$

Where i represents the layer and a, b and n the flow periods.

$$P_D' = \frac{\partial P_D}{\partial Int_D} \qquad (8)$$

P_D for early radial flow period common to both no crossflow and crossflow layered systems was represented by equation (9)

$$P_D = \frac{\alpha}{8L_D}\sqrt{\frac{K}{K_y}}Ei\left(-\frac{r_{WD}{}^2}{4\tau_D}\right)$$

(9)

2.2.1 No crosssflow layered system

P_D for intermediate and late time flow periods of layer one given in equations (10) and (11)

$$P_D = \frac{2\pi}{X_{eD}Y_{eD}}\int_{t_{Ds1}}^{t_D}\left\{\begin{matrix}\left(1+\frac{4X_{eD}}{\pi}\sum_{m=1}^{\infty}\exp\left[--\frac{m^2\pi^2\tau_D}{X_{eD}{}^2}\right]\sin\frac{m\pi x_f}{2X_{eD}}\cos\frac{m\pi x_{wD1}}{X_{eD}}\cos\frac{m\pi x_{D1}}{X_{eD}}\right). \\ \left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\pi^2\tau_D}{Y_{eD}{}^2}}\cos\frac{n\pi y_{wD1}}{Y_{eD}}\cos\frac{n\pi Y_{D1}}{Y_{eD}}\right). \\ \left(\sum_{i=1}^{\infty}\exp\left[--\frac{(2i-1)^2\pi^2\tau_D}{4h_D{}^2}\right]\sin\frac{(2i-1)\pi z_{wD1}}{2h_D}\sin\frac{(2i-1)\pi z_{D1}}{2h_D}\right)\end{matrix}\right\}\partial\tau \qquad (10)$$

$$P_D = \frac{2\pi}{X_{eD}Y_{eD}}\int_{t_{Ds1}}^{t_D}\left\{\begin{matrix}\left(1+\frac{4X_{eD}}{\pi}\sum_{m=1}^{\infty}\exp\left[--\frac{m^2\pi^2\tau_D}{X_{eD}{}^2}\right]\sin\frac{m\pi x_f}{2X_{eD}}\cos\frac{m\pi x_{wD2}}{X_{eD}}\cos\frac{m\pi x_{D2}}{X_{eD}}\right). \\ \left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\pi^2\tau_D}{Y_{eD}{}^2}}\cos\frac{n\pi y_{wD2}}{Y_{eD}}\cos\frac{n\pi Y_{D2}}{Y_{eD}}\right). \\ \left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\pi^2\tau_D}{h_D{}^2}}\cos\frac{n\pi z_{wD2}}{h_D}\cos\frac{n\pi z_{D2}}{h_D}\right)\end{matrix}\right\}\partial\tau \qquad (11)$$

2.2.2 Crossflow layered system

Owing to the permeable interface in the crossflow layered reservoir system, the following additional parameters are required for proper modelling of the system.

Modification factor (E_J)

$$E_J = \frac{W_1 + W_2}{2\pi\left(W_1^2 + W_2^2\right)} \qquad (12)$$

Where W_1 and W_2 represent P_D of layer one and layer two of the system.

Total pay thickness (h_{Dt}) for the system

$$h_{Dt} = h_{D1} + h_{D2} \qquad (13)$$

Time normalisation factor or crossflow coefficient (β)

$$\beta = \frac{\phi_i \mu_i C_{ti} L_i^{\,2} K_{i+1}}{\phi_{i+1} \mu_{i+1} C_{ti+1} L_{i+1}^{\,2} K_i} \qquad (16)$$

P_D for intermediate and late time flow periods in layer one are determined by equations (14) and (15)

$$P_{D1} = \sqrt{\pi}\beta E_2 \int_{t_{De1}}^{t_{De2}} \left\{ \begin{array}{l} erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}} + X_{D2}}{2\sqrt{\beta\tau_D}}\right) + erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}} - X_{D2}}{2\sqrt{\beta\tau_D}}\right). \\ \left(1 + 2\sum_{n=1}^{\infty} \exp^{-\frac{n^2\pi^2\beta\tau_D}{Y_{eD}^{2}}} \cos\dfrac{n\pi y_{wD2}}{Y_{eD}} \cos\dfrac{n\pi Y_{D2}}{Y_{eD}}\right) \\ \dfrac{1}{2\sqrt{\beta\tau_D}} \exp^{-\frac{(Z_{D2} - Z_{WD2})^2}{4\beta\tau_D}}. \end{array} \right\} \partial\tau \qquad (14)$$

$$P_{D1} = \frac{4\pi E_1}{X_{eD} Y_{eD}} \int_{t_{De3}}^{t_D} \left\{ \begin{array}{l} \left(1 + \dfrac{4X_{eD}}{\pi} \sum_{m=1}^{\infty} \exp^{\left[-\frac{m^2\pi^2\tau_D}{X_{eD}^{2}}\right]} \sin\dfrac{m\pi x_f}{2X_{eD}} \cos\dfrac{m\pi x_{wD1}}{X_{eD}} \cos\dfrac{m\pi x_{D1}}{X_{eD}}\right). \\ \left(1 + 2\sum_{n=1}^{\infty} \exp^{-\frac{n^2\pi^2\tau_D}{Y_{eD}^{2}}} \cos\dfrac{n\pi y_{wD}}{Y_{eD}} \cos\dfrac{n\pi Y_W}{Y_{eD}}\right). \\ \left(\sum_{n=1}^{\infty} \exp^{\left[-\frac{n^2\pi^2\tau_D}{h_{D1}^{2}}\right]} \sin\dfrac{n\pi Z_{wD1}}{h_{D1}} \sin\dfrac{n\pi z_{D1}}{h_{D1}}\right) \end{array} \right\} \partial\tau \qquad (15)$$

P_D for intermediate and late time flow periods in layer two are determined by equations (17) and (18)

$$P_{D2} = \sqrt{\pi}\beta E_2 \int_{t_{De1}}^{t_{De2}} \left\{ \begin{array}{l} erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}} + X_{D2}}{2\sqrt{\beta\tau_D}}\right) + erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}} - X_{D2}}{2\sqrt{\beta\tau_D}}\right). \\ \left(1 + 2\sum_{n=1}^{\infty} \exp^{-\frac{n^2\pi\beta\tau_D}{Y_{eD}^{2}}} \cos\dfrac{n\pi y_{wD2}}{Y_{eD}} \cos\dfrac{n\pi Y_{D2}}{Y_{eD}}\right) \\ \dfrac{1}{2\sqrt{\beta\tau_D}} \exp^{-\frac{(Z_{D2} - Z_{WD2})^2}{4\beta\tau_D}}. \end{array} \right\} \partial\tau \qquad (17)$$

$$P_{D2} = \frac{4\pi\beta E_2}{X_{eD} Y_{eD}} \int_{t_{De3}}^{t_D} \left\{ \begin{array}{l} \left(1 + \dfrac{4X_{eD}}{\pi} \sum_{m=1}^{\infty} \exp^{\left[-\frac{m^2\pi^2\beta\tau_D}{X_{eD}^{2}}\right]} \sin\dfrac{m\pi x_f}{2X_{eD}} \cos\dfrac{m\pi x_{wD2}}{X_{eD}} \cos\dfrac{m\pi x_{D2}}{X_{eD}}\right). \\ \left(1 + 2\sum_{n=1}^{\infty} \exp^{-\frac{n^2\pi^2\beta\tau_D}{Y_{eD}^{2}}} \cos\dfrac{n\pi y_{wD2}}{Y_{eD}} \cos\dfrac{n\pi Y_{D2}}{Y_{eD}}\right). \\ \left(\sum_{n=1}^{\infty} \exp^{\left[-\frac{(2n-1)^2\pi^2\beta\tau_D}{4h_{D2}^{2}}\right]} \sin\dfrac{(2n-1)\pi z_{wD2}}{2h_{D2}} \sin\dfrac{(2n-1)\pi z_{D2}}{2h_{D2}}\right) \end{array} \right\} \partial\tau \qquad (18)$$

2.3 Program description

The program was designed and implemented with the aid of Visual Basic Programming language. This robust programming tool enables the equations involved in the layered mathematical models to be coded. The programming process involved three main steps.

2.3.1 Input stage

Several graphical user interfaces (GUI's) were designed with forms, labels, textboxes and command buttons. The user chooses the type of reservoir, then he inputs well, fluid and reservoir data(in S.I units) necessary for the determination of the P_D.

2.3.2 Analysis

The parameters are converted to their dimensionless form and used in the computations. The model equations contained complex summation and integrals that can only be handled numerically. The pressure response for the early time flow period was computed using the exponential integral solution and Gauss Laguerre numerical method. The Gauss lagrange method was employed to determine the dimensionless pressure for the other flow periods [16,17]. Total P_D for the system was assembled using the principle of superposition in time. The solution was processed speedily within seconds as a result of the robust codes used in programing the reservoir model. Flow chart of the program in figure 1 shows the steps involved in the analysis.

2.3.3 Output stage

Results for the computation of pressure and derivatives of the layered reservoir were displayed as numeric values in a data grid and also with an option of graphical representation on Cartesian scale plotting the log of dimensionless time (log t_D) against dimensionless pressure(logP_D)

3. Program Algorithm

The following is the software's algorithm

Start
Select type of reservoir interface.
Option 1: No-crossflow layered reservoir
Option2: Crossflow layered reservoir
if option value =1 **then if** reservoir type is no-crossflow **then**
If desired parameter = P_D **Then** Input known parameters **L, ,h, k, z_w, r_w, ϕ, µ**
Compute flow periods t_{DE1}, t_{DE2}, t_{DE3}, t_{D4} . **If** dimensionless time t_D < first flow period T_{DE1} **Then** Infinite acting flow period, Exponential integral Equation
Compute P_{D1} using the Laguerre quadrature **End If**
Else If t_{DE3} > third flow period t_{DE3} **Then** Compute P_{DE3} using Legendre quadrature
P_D=P_{D1}+P_{D2}+P_{D3}+...... **End If.** **If** option's value's=2 **Then**
If reservoir is crossflow systems **Then** Input known values of L, r_W, h, z_w, k_y, k_x, k_z
Compute BTA, H_{DT}, K, E
Compute flow periods t_{DE1}, t_{DE2}, t_{DE3}, t_{D4} .
Compute P_{D1} , P_{D2} , P_{D3} , P_{D4} using same method as no crossflow reservoir
End IF Compute Pressure derivative using differentiation
Else If t_D > second flow period T_{DE2} **Then** Compute P_{D2} using Lagendre quadrature
Plot graph
End

4. Results and Discussion

Numeric examples (Tables 1 and 2) were employed to demonstrate the relevance and robustness of the software. The following well/fluid properties were assumed constant for both (crossflow and no crossflow) reservoir system. Wellbore radius (r_w) = 0.375ft, total compressibility (C_t) = 3.0E-6psi^{-1}, Porosity(φ) = 1%, Viscosity (µ) =1Cp. Using the data from table one, values of pressure and derivative of horizontal wells situated in each layer were generated. The results(increasing pressure and derivative values with time)for layer one indicates that the top layer(layer 2) behaves like a reservoir with no flow boundaries having no communication with the upper layerbecause the interface is sealed

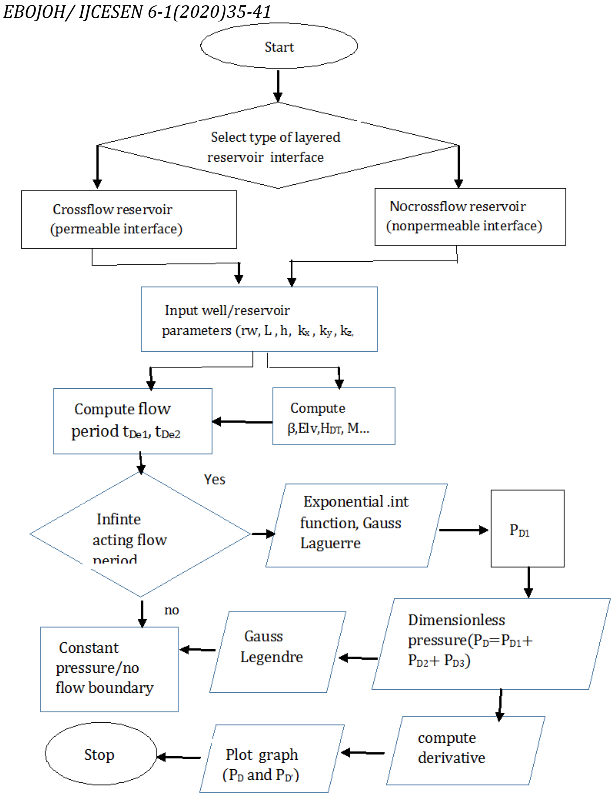

Figure 1: *Flowchart for the computer program.*

(nocrossflow).Pressure and derivative response from layer one depicts the trend of a reservoir with a bottom waterdrive (constant pressure boundary) as pressure values increase and becomes constant while the derivative display initial constant values at early time with a collapse to zero at late time. Figures (2)-(5) clearly depicts the results for no crossflow system. For crossflow system, same procedure for computing pressure and derivative was adopted. The program computed the flow periods that may occur, crossflow coefficients, mo dification factor, total pay thickness and total permeability of the system. So as to reduce congestion on the crossflow

Table. 1 Showing well/reservoir data for No-Crossflow reservoir system.

Parameter	Value
L, ft	1000
h, ft	200
Z_w, ft	100
X_e, ft	6 000
Y_e, ft	6 000
Elv, ft	20
K_i, md	1:1:1

Table. 2 Showing well/reservoir data for Crossflow reservoir system.

Parameter	Layer 1	Layer 2
L, ft	1000	1000
h, ft	100	100
Z_w, ft	50	50
X_e ,ft	16 000	16000
Y_e, ft	10 000	10 000
Elv, ft	10	10
K_i, md	1:1:1	10:10: 10

reservoir form. These results can be viewed from the immediate window. Results were also displayed in a datagrid shown in figures (6)-(9). Layer one and two display similar trend for pressure and derivative. Similarity in behavoir can be traced to the permeable interface, There is communication(crossflow) between the layers, pressure values increased with time and stabilized at late time while derivatve was initially constant but collapsed to zero at latetimes. Both layers behave like reservoirs that have a constant pressure boundary. For proper visual appreciation and interpretation of results cartesian plots of the results were provided. The usefulness and effectiveness of the computer programs can be seen clearly. All aspect of pressure transient test analysis: data preparation, model identification, parameter calculation, and model validation and presentation of results have benefited because of the use of computer[18, 19]

5. Conclusion

This work has presented models for a two layered reservoir with horizontal wells, also formulated and presented a computer program to generate dimensionless pressure and derivative values of the reservoir system with a bottom water drive. User friendly GUI's were designed for input of data and output of results. From the results shown in figure 2-5 it was observed that the pressure and derivative response of no crossflow reservoir system was higher, layer 1 displayed the behaviour of a bottom water drive reservoir while layer 2 showed the characteristics of a bounded reservoir whose lateral boundaries are infinite. Figure 6-9 showed screenshots of GUI's for the crossflow reservoir. Similarity in flow behaviour for both layers was observed, at early time pressure values increased and became constant at late times while derivative values became constant and collapsed to zero. This response is typical of that of reservoir with a

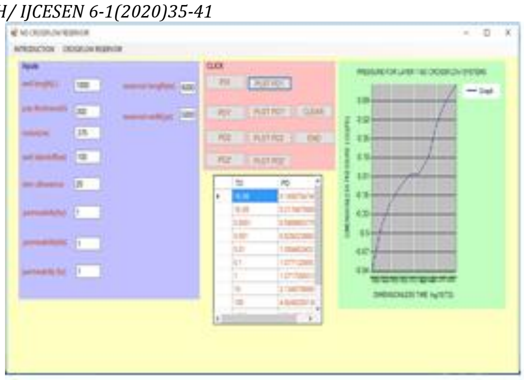

Figure 2: *Screenshot of result for (no crossflow reservoir) layer 1 dimensionless pressure computations and graph plot*

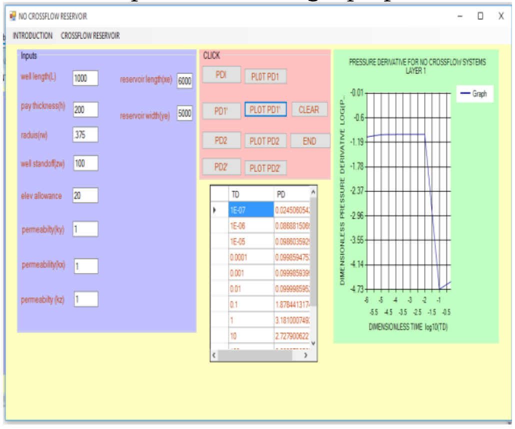

Figure 3: *Screenshot of result for (no flow reservoir) layer 1 dimensionless pressure derivative and graph plot*

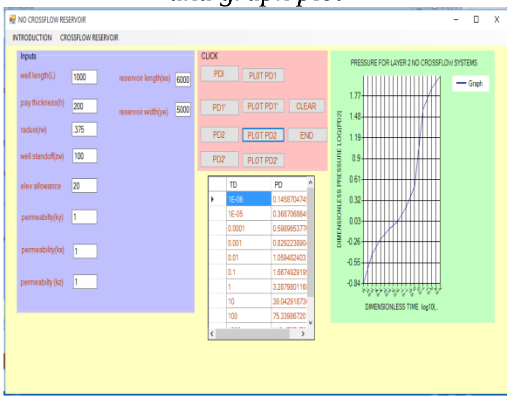

Figure 4: *Screenshot of result for (no crossflow reservoir) layer 2 dimensionless pressure computations and graph plot.*

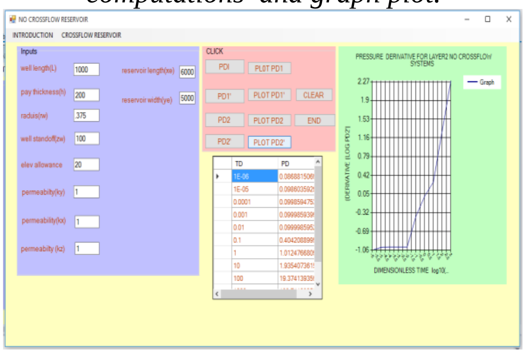

Figure 5: *Screenshot of result for (no crossflow reservoir) layer 2 dimensionless pressure derivative and graph plot*

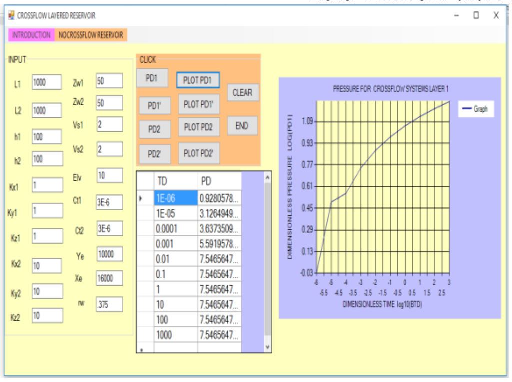

Figure 6: Screenshot of results for (Crossflow reservoir) layer 1 dimensionless pressure computations and graph plot.

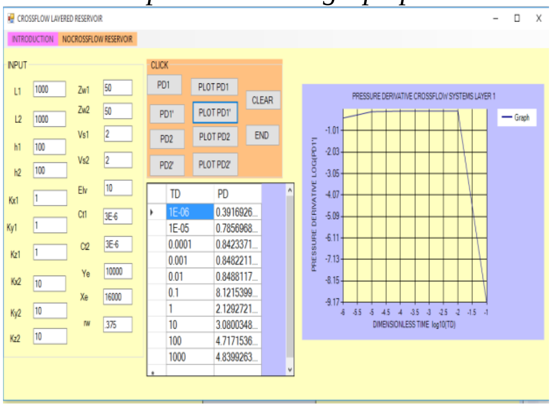

***Figure 7:** Screenshot of result for (Crossflow reservoir) layer 1 dimensionless pressure derivative and graph plot*

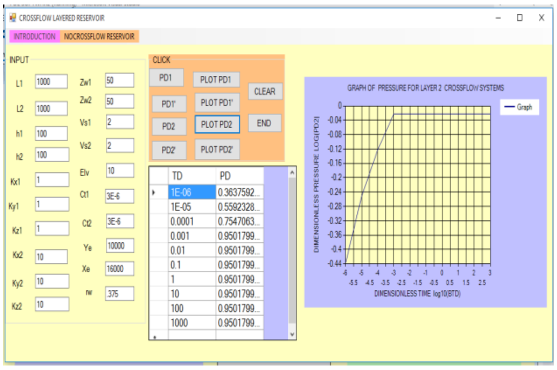

***Figure 8:** Screenshot of results for (Crossflow reservoir) layer 2 dimensionless pressure computations and graph plot*

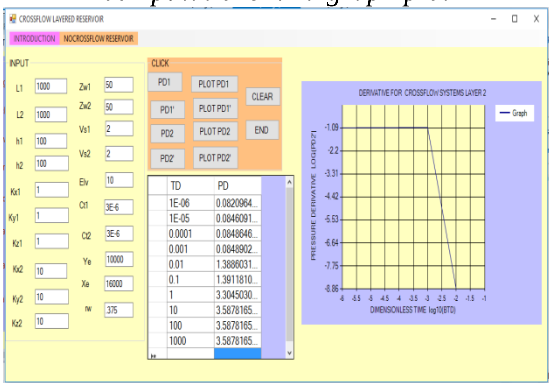

***Figure 9**: Screenshot of results for Crossflow reservoir layer 2 dimensionless pressure derivative and graph plot.*

constant pressure boundary with the more permeable layer (layer 2) having a lower pressure and derivative response. The graph of the results further highlights the capability of the computer program in model identification and reservoir characterization. Results from this study will improve the quality of test analysis of layered reservoir system with bottom water drive using horizontal wells, especially in the areas of drawdown, build up and interference test. It will also serve as an effective learning tool because it's easy and fast and results have acceptable level of accuracy.

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Nomenclature

C	Compressibility	psi^{-1}
Elv	Elevation allowance,	ft
h	Pay thickness,	ft
i	Distance in x, y or z direction,	ft
K	Permeability,	md
L	Well length,	ft
P	Pressure,	psi
q	flow rate,	STB/D
r	well radius,	ft
t	time,	hr
X	distance in the x-axis,	ft
Y	distance in the y-axis,	ft
Z	distance in the z-axis,	ft
Greek letters		
Δt	time increment,	hr
π	Pi	
Φ	porosity, fraction	
β	crossflow coefficient	
µ	viscosity	cp
α	constant	
τ	integration variable	
Subscript		
D	Dimensionless	
e	External	
t	Total	
w	Wellbore	