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The Soft Quark Dispersion Relations at Next-to-Leading Order in Hot QCD

K. BOUAKAZ[1*], A. ABADA[1,2] and K. BENCHALLAL[1]

[1] Physics Department, Laboratoire de Physique des Particules et de Physique Statistique, Ecole Normale Supérieure, BP 92 Vieux-Kouba, Algiers 16050 Algeria
[2] Physics Department, United Arab Emirates University, POB 17551 Al Ain, United Arab Emirates.

* Corresponding Author : kbouakaz@yahoo.fr

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Abstract: We give a compact analytic expression for the fully-dressed next-to-leading order contribution to the retarded quark self-energy in the context of hard-thermal-loop summed perturbation of massless QCD at high temperature. We also give the analytic expressions of the hard thermal loop vertex functions. The calculation is done using the real-time formalism.

1. Introduction

Over the last decade, a great deal of work has been devoted to the study of field theories at finite temperature and density. This intense effort has been partly motivated by the hypothetical existence of a quark-gluon plasma (QGP) phase at very high temperature and/or density. This QGP phase is connected to the early stages of the evolution of the universe [1] and may also be observable in relativistic heavy-ion collisions [2].
Among the important theoretical issues at stake is the strong infrared behavior of gauge theories at high temperature which generally leads to a number of paradoxes in perturbation theory [3]. In particular, an apparent gauge dependence of physical quantities has been found in certain calculations which lead to a controversy regarding their interpretation. An important example is the gluon damping rate at zero momentum in one-loop calculations: this quantity was found positive in the time-like axial and Coulomb gauges [4,5,6] and negative in covariant and background field gauges [7,8]. The problem was resolved in [9]. Indeed, it turns out that in order to calculate consistently at high temperature, we have to use an effective perturbation that sums the so-called hard thermal loops (HTL) into dressed propagators and vertices [9, 10]. When applied to the zero-momentum gluon damping rate, the effective perturbation yields a finite, positive, and gauge-independent result [11].
Pushing the HTL-summed perturbation theory further, the infrared behavior of the gluon and quark damping rates have been studied in [12,13,14,15,16,17,18] using the imaginary time formalism. A similar calculation has also been carried out in the context of scalar electrodynamics [19].
In the present communication, we show the main steps leading to the determination of an analytic expression for the next-to-leading contribution to the retarded quark self-energy in massless QCD at high temperature using the real-time formalism [20, 21, 22].

2. Effectvie Expansion

We use covariant gauge. The HTL-summed fermion propagator is defined as follows:

$$ {}^{*}\Delta_F(P) = i\left(\not P - \delta\Sigma_{htl}(P)\right)^{-1} , \qquad (1) $$

$\delta\Sigma_{htl}(P)$ in this expression is the hard-thermal-loop contribution to the fermion self-energy. The HTL-summed gluon propagator is given by:

$$ {}^{*}\Delta^{\mu\nu}_{ra/ar}(K) = P_T^{\mu\nu}\frac{1}{\delta\Pi_T^{r/a} - K^2 \mp i\,\mathrm{sgn}(k_0)\varepsilon} + P_L^{\mu\nu}\frac{1}{\delta\Pi_L^{r/a} - K^2 \mp i\,\mathrm{sgn}(k_0)\varepsilon} + D^{\mu\nu}\frac{\xi}{K^2 \mp i\,\mathrm{sgn}(k_0)\varepsilon} \qquad (2) $$

$P_T^{\mu\nu}, P_L^{\mu\nu}, D^{\mu\nu}$ are the projection tensors.

The HTL-summed fermion-gauge-boson vertex is given by:

$$^{*}\Gamma^{\mu} = \gamma^{\mu} + \delta\Gamma^{\mu} \;, \tag{3}$$

in which the quantity $\delta\Gamma^{\mu}$ is the HTL contribution. Also, The HTL-summed fermion-pair 2-gauge boson vertex is:

$$^{*}\Gamma^{\mu\nu} = \delta\Gamma^{\mu\nu} \;, \tag{4}$$

where $\delta\Gamma^{\mu\nu}$ is the corresponding HTL contribution.

The quark dispersion relations are determined by the poles of the HTL-summed quark propagator or, equivalently, are found as solutions of the equation:

$$\det\left(P\!\!\!/ - \Sigma_{ret}(P)\right) = 0, \tag{5}$$

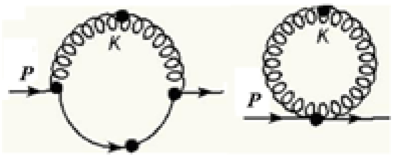
The next-to leading order quarks energy and damping rate are therefore given by:

$$\omega_{\pm}^{(1)}(p) = \left.\frac{\mathrm{Re}\,^{*}\Sigma_{\pm}(\omega, p)}{1 - \dfrac{\partial \Sigma_{\pm}^{htl}(\omega, p)}{\partial\omega}}\right|_{\omega=\omega_{\pm}(p)} ;$$

$$\gamma_{\pm}(p) = -\left.\frac{\mathrm{Im}\,^{*}\Sigma_{\pm}(\omega, p)}{1 - \dfrac{\partial \Sigma_{\pm}^{htl}(\omega, p)}{\partial\omega}}\right|_{\omega=\omega_{\pm}(p)} , \tag{6}$$

In these expressions, $\omega_{\pm}(p)$ is the leading-order quark energy. Thus, to obtain the next-to leading-order dispersion relations for soft quarks, we have to determine the next-to leading-order quark self-energy.

The diagrams that contribute to the next-to-leading-order quark self-energy are the following two diagrams with soft internal loop momentum.



Dressed quark self energies

The contribution of the first diagram in the Keldysh basis is given by:

$$^{*}\Sigma^{ab}(P)\Big|_{diagram\;\;1} = g^{2}Tr\left[^{*}\Gamma_{\mu}^{\alpha\alpha\beta}(P,Q)\,^{*}\Delta_{F}^{\beta'\beta}(Q) \times\,^{*}\Gamma_{\mu}^{\beta'\alpha'b}(Q,P)\,^{*}\Delta_{\mu\nu}^{\alpha\alpha'}(K)\right]; \tag{7}$$

and the contribution of the second diagram is:

$$^{*}\Sigma^{ab}(P)\Big|_{diagram\,2} = g^{2}Tr\left[^{*}\Gamma_{\mu\nu}^{abcd}(P,K,-K,-P)\,^{*}\Delta_{\mu\nu}^{cd}(K)\right], \tag{8}$$

As found in [23] and using the notation there, the contributions of the two diagrams are grouped in the following two terms:

$$\begin{aligned}
T_1 = \sum_{\sigma=\pm}\int dK\Big\{&\left(d_T(K)N_B(k_0) + {}^{*}\Delta_T^{adv}(K)N_F(q_0)\right)\Delta_{\sigma}^{ret}(Q)\\
&\times\Big(2\,{}^{*}\Gamma_{\mu}^{0}[2](P,Q)\,{}^{*}\Gamma^{0\mu}[2](P,Q) - \left({}^{*}\Gamma_{\mu}^{\lambda}[2](P,Q)\right)^2\\
&-\sigma\left({}^{*}\Gamma_{\mu}^{0}[2](P,Q)\,{}^{*}\Gamma^{i\mu}[2](P,Q) + {}^{*}\Gamma_{\mu}^{i}[2](P,Q)\,{}^{*}\Gamma^{0\mu}[2](P,Q)\right)\hat{q}^i\\
&-\left({}^{*}\Gamma_{\mu}^{i}[2](P,Q)\,{}^{*}\Gamma^{0\mu}[2](P,Q) + {}^{*}\Gamma_{\mu}^{0}[2](P,Q)\,{}^{*}\Gamma^{i\mu}[2](P,Q)\right)\hat{p}^i\\
&+\sigma\Big({}^{*}\Gamma_{\mu}^{i}[2](P,Q)\,{}^{*}\Gamma^{j\mu}[2](P,Q) + {}^{*}\Gamma_{\mu}^{j}[2](P,Q)\,{}^{*}\Gamma^{j\mu}[2](P,Q)\\
&\quad - g^{ij}\,{}^{*}\Gamma_{\mu}^{\lambda}[2](P,Q)\,{}^{*}\Gamma_{\mu}^{\lambda}[2](P,Q)\Big)\hat{p}^i\hat{q}^j - {}^{*}\Delta_T^{adv}(P)\Delta_{\sigma}^{adv}(R)N_F(q_0)\\
&\times\Big(2\,{}^{*}\Gamma_{\mu}^{0}[5](P,Q)\,{}^{*}\Gamma^{0\mu}[5](P,Q) - {}^{*}\Gamma_{\mu}^{\lambda}[5](P,Q)\,{}^{*}\Gamma_{\lambda}^{\mu}[5](P,Q)\Big)\\
&-\sigma\left({}^{*}\Gamma_{\mu}^{0}[5](P,Q)\,{}^{*}\Gamma^{i\mu}[5](P,Q) + {}^{*}\Gamma_{\mu}^{i}[5](P,Q)\,{}^{*}\Gamma^{0\mu}[5](P,Q)\right)\hat{q}^i\\
&-\left({}^{*}\Gamma_{\mu}^{i}[5](P,Q)\,{}^{*}\Gamma^{0\mu}[5](P,Q) + {}^{*}\Gamma_{\mu}^{0}[2](P,Q)\,{}^{*}\Gamma^{i\mu}[5](P,,Q)\right)\hat{p}^i\\
&+\sigma\Big({}^{*}\Gamma_{\mu}^{i}[5](P,Q)\,{}^{*}\Gamma^{j\mu}[5](P,Q) + {}^{*}\Gamma_{\mu}^{j}[5](P,Q)\,{}^{*}\Gamma^{j\mu}[5](P,Q)\\
&\quad - g^{ij}\,{}^{*}\Gamma_{\mu}^{\lambda}[5](P,Q)\,{}^{*}\Gamma_{\mu}^{\lambda}[5](P,Q)\Big)\hat{p}^i\hat{q}^j\Big\},
\end{aligned}$$

and

$$\begin{aligned}
T_2 = \sum_{\sigma=\pm}\int dK\Big\{&\left(\left(\tilde{d}_T(K) + \tilde{d}_L(K)\right)N_B(k_0) + \left({}^{*}\tilde{\Delta}_T^{adv}(K) + {}^{*}\tilde{\Delta}_L^{adv}(K)\right)N_F(q_0)\right)\\
&\times\Big(2\left({}^{*}\Gamma_{00}[2](P,Q)\right)^2 - \left({}^{*}\Gamma_{\tau 0}[2](P,Q)\right)^2\\
&-2\sigma\,{}^{*}\Gamma_{00}[2](P,Q)\,{}^{*}\Gamma_{i0}[2](P,Q)\hat{q}^i - 2\,{}^{*}\Gamma_{00}[2](P,Q)\,{}^{*}\Gamma_{0i}[2](P,Q)\hat{p}^i\\
&+\sigma\left(2\,{}^{*}\Gamma_{i0}[2](P,Q)\,{}^{*}\Gamma_{j0}[2](P,Q) - g^{ij}\left({}^{*}\Gamma_{\tau 0}[2](P,Q)\right)^2\right)\hat{p}^i\hat{q}^j\\
&-2\left({}^{*}\tilde{\Delta}_T^{adv}(K) + {}^{*}\tilde{\Delta}_L^{adv}(K)\right)N_F(q_0)\Big(2\left({}^{*}\Gamma_{00}[5](P,Q)\right)^2 - \left({}^{*}\Gamma_{\tau 0}[5](P,Q)\right)^2\\
&-2\sigma\,{}^{*}\Gamma_{00}[5](P,Q)\,{}^{*}\Gamma_{i0}[5](P,Q)\hat{q}^i - 2\,{}^{*}\Gamma_{00}[5](P,Q)\,{}^{*}\Gamma_{0i}[5](P,Q)\hat{p}^i\\
&+\sigma\left(2\,{}^{*}\Gamma_{i0}[5](P,Q)\,{}^{*}\Gamma_{j0}[5](P,Q) - g^{ij}\left({}^{*}\Gamma_{\tau 0}[5](P,Q)\right)^2\right)\hat{p}^i\hat{q}^j,
\end{aligned}$$

In these expressions, $\int dK = \frac{g^2}{4\pi^3}\int dk_0\int k^2 dk\int\frac{d\Omega}{4\pi}$, $d_T(K) = D_T^{ret}(K) - D_T^{adv}(K)$ and $^{*}\Gamma_{\tau\mu}[i](P,Q) = g_{\tau\mu} + \Gamma_{\tau\mu}^{htl}(P,Q)$; $i = 2 \text{ or } 5$

We have expressed the effective quark self-energy in a compact form. Next is to give the analytic expressions of the vertex functions involved.

3 The Vertex Functions

As a prototype, consider the function:

$$\delta\Gamma^{\mu\nu}(P,P') = \int\frac{d\Omega}{4\pi}\frac{\hat{K}^{\mu}\hat{K}^{\nu}}{\left(\hat{K}.P + i\varepsilon\right)\left(\hat{K}.P' - i\varepsilon\right)} \tag{11}$$

We first express $\delta\Gamma^{00}$ using the Feynman parameterization:

$$\delta\Gamma^{00}(P,P') = \frac{-m_f^2}{4\pi}\int_0^1 ds\int d\Omega\frac{-1}{\left(s\left(\hat{K}.P + i\varepsilon\right) - (1-s)\left(\hat{K}.P' - i\varepsilon\right)\right)^2} \tag{12}$$

The denominator can be written as $D = \left(\hat{K}.R(s) + i\varepsilon\right)^2$, $R(s) = -P' + sQ$ $\quad Q = P + P'$

The integration over $d\Omega$ gives the following result:

$$\delta\Gamma^{00}(P,P') = \frac{1}{2}\int_0^1\frac{ds}{r(s)}\frac{2r}{\left(r_0 + i\varepsilon\right)^2 - r^2}$$

(13)

We can write

$$(r_0 + i\varepsilon)^2 - r^2 = As^2 + 2Bs + C \quad A = Q^2,$$
$$B = -QP' + i\varepsilon q_0,\ C = (p_0' - i\varepsilon)^2 - p'^2$$

We then find the result:

$$\delta\Gamma^{00}(P,P') = \frac{1}{2\sqrt{\Delta'}}(F + jG), \quad \Delta' = B^2 - AC \qquad (14)$$

with $F = \frac{1}{2} ln \frac{(1-r_1)^2 + r_1^2}{r_1^2 + i_1^2} - \frac{1}{2} ln \frac{(1-r_2)^2 + r_2^2}{r_2^2 + i_2^2}$

and $G = \arctan\frac{1-r_1}{i_1} + \arctan\frac{r_1}{i_1} - \arctan\frac{1-r_2}{i_2} - \arctan\frac{r_2}{i_2},$

Here, the quantities r_1, r_2 are the real parts of the root of denominator, and i_1, i_2 their imaginary parts. Following the same steps, we derive the other components of the vertex functions. We have:

$$\delta\Gamma^{0i}(P,P') = \int_0^1 ds \int \frac{d\Omega}{4\pi} \frac{-k^0 \hat{k}^i}{\left(s(\hat{K}.P + i\varepsilon) - (1-s)(\hat{K}.P' - i\varepsilon)\right)^2}$$
$$= \frac{k^0}{2} \frac{\hat{r}^i}{r} \int_0^1 ds \left(\frac{2(r_0 + i\varepsilon)}{(r_0 + i\varepsilon)^2 - r^2} + \frac{1}{2r} \ln \frac{2(r_0 + r)^2 + \varepsilon^2}{(r_0 - r)^2 + \varepsilon^2} \right.$$
$$\left. - \frac{i}{r} \left\{ \arctan\frac{r_0 + r}{\varepsilon} - \arctan\frac{r_0 - r}{\varepsilon} \right\} \right),$$

$$\delta\Gamma^{ij}(P,P') = \int_0^1 ds \int \frac{d\Omega}{4\pi} \frac{-\hat{k}^i \hat{k}^j}{\left(s(\hat{K}.P + i\varepsilon) - (1-s)(\hat{K}.P' - i\varepsilon)\right)^2} = \frac{1}{2} \int_0^1 ds \left(\alpha \hat{r}^i \hat{r}^j + \beta g^{ij}\right),$$
$$\alpha = \frac{1}{r^2} \left(3 - 3(r_0 + i\varepsilon) \left\{ \frac{1}{2r} \ln \frac{(r_0 + r)^2 + \varepsilon^2}{(r_0 - r)^2 + \varepsilon^2} - \frac{i}{r} \left(\arctan\frac{r_0 + r}{\varepsilon} - \arctan\frac{r_0 - r}{\varepsilon} \right) \right\} \right.$$
$$\left. + \frac{3(r_0 + \mathrm{i}\varepsilon)^2 - r^2}{(r_0 + \mathrm{i}\varepsilon)^2 - r^2} \right)$$
$$\beta = \frac{-1}{r^2} \left(2 - (r_0 + i\varepsilon) \left\{ \frac{1}{2r} \ln \frac{(r_0 + r)^2 + \varepsilon^2}{(r_0 - r)^2 + \varepsilon^2} - \frac{i}{r} \left(\arctan\frac{r_0 + r}{\varepsilon} - \arctan\frac{r_0 - r}{\varepsilon} \right) \right\} \right)$$

The integration over s is to be carried out numerically.

3. Conclusions

We have given a compact analytic expression for the complete next-to-leading contribution to the retarded quark self-energy in the context of hard-thermal-loop summed perturbation of QCD at high temperature. We have also determined the compact expressions for the hard thermal loop contributions to the vertex functions. What remains to do is to perform the integrals numerically to evaluate the quark energies and damping rates. These integrations are not straightforward and will necessitate heavy numerical work. This is currently in progress.

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