International Journal of Computational and Experimental Science and ENgineering
(IJCESEN)
Vol. 9-No.2 (2023) pp. 123-132
http://dergipark.org.tr/en/pub/ijcesen

ISSN: 2149-9144

Research Article

Applying the Fuzzy PERT Method in Project Management: A Real-Life Case Study

Emre KENAR[1], Mümtaz İPEK[2], Muharrem DÜGENCİ[3], Ömer Abdulaziz KORKMAZ[4]*

[1]Karabük University, Faculty of Engineering, Industrial Engineering Department, 78050, Karabük-Turkey
Email: emrekenar06@gmail.com - **ORCID:** 0000-0002-0989-3242

[2]Sakarya University, 54050, Sakarya-Turkey
Email: ipek@sakarya.edu.tr - **ORCID:** 0000-0001-9619-2403

[3]Karabük University, Faculty of Engineering, Industrial Engineering Department, 78050, Karabük-Turkey
Email: mdugenci@karabuk.edu.tr - **ORCID:** 0000-0001-7091-9096

[4]Karabük University, Faculty of Engineering, Industrial Engineering Department, 78050, Karabük-Turkey
* **Corresponding Author**: **Email:** o_aziz96@outlook.com - **ORCID:** 0000-0002-4603-0876

Article Info:

DOI: 10.22399/ijcesen.1262975
Received : 10 March 2023
Accepted : 10 June 2023

Keywords

Project Management
PERT
Fuzzy PERT
Fuzzy Logic

Abstract:

Time is the most precious resource for any business simply because no one can bring time back. Therefore, there is a huge need to be able to control time accurately. Also, the competitiveness between companies increased the value of time. Many investments were allocated to build methodologies that help to use time effectively. Because it became a fact that the speed of work accomplishment is the main factor for being a considerable competitor. Planning for future projects usually relays on non-fixed but mostly expected data. It is more reliable to use fuzzy methods for this situation. In this study, Fuzzy PERT (FPERT) was used as a project management technique to plan the construction of a marble factory from the establishment phase to the plan of the machines that will be used later on. FPERT showed more realistic results than any classical method would. Out of 16 routes, the final result of the completion time was (90.9,122.1,151.9) days which was determined after triangular fuzzy numbers for the activity times in the entire project. This article aims to prove the efficiency of fuzzy methodologies especially FPERT to be conducted in future planned projects through this real-life case study.

1. Introduction

Recently, the competition between companies forced all the owners and managers to develop their firms. Technology is a double-edged sword in this case because it is one of the most significant reasons for to increase in the competition rate between companies, and it is the solution using it to be improved. The main aim for all firms is to use all their resources (human & material) effectively. They seek to be more productive, complete their projects and achieve their targets without any waste and with the least costs. The most important resource that companies should focus on is time. By managing time appropriately, following up with the other sources can happen very smoothly. Therefore, there becomes a huge need for new models in order to manage complicated projects and operations' time to simplify tracking and following up. Both CPM (Critical Path Method) and PERT (Program Evaluation and Review Technique) were developed based on the network to help project managers to schedule projects [1]. They were the most popular models that companies tend to use due to their efficiency in managing projects and simplifying the timeline of each process.

CPM and PERT were developed in the same period between 1956 and 1958 but by two different research groups. CPM was developed while building a chemical factory by the R&D department in E. I. Dupont De Nemours company. On the other hand, the PERT model was developed while conducting Polaris nuclear submarine project for the US Navy by Booz Allen Hamilton. As a result, by applying PERT, the US Navy could finish the project two years before the scheduled time. By applying techniques such as CPM and PERT, it became easier to observe the important activities and operations that affect the project to focus on them and provide the critical resources to be able to schedule the project precisely. At the same time, the activities that could have less importance or efficiency on the project schedule can be postponed or neglected to finish the project on time. Nevertheless, the classical methods that gave very good results in a simple and isolated environment, may not give the same outcomes in complex business. As a matter of fact, the rapid improvement of science and technology makes the process of decision-making more difficult owing to the uncertainty and the difficulties of analyzing features. Therefore, modern business society became more complex, and the classical models need to be improved. The classical PERT model in the modern complex environment couldn't be accurate so the duration variance in projects in real-life became overestimated and as a result of these criticisms, many interesting and extensive studies had been conducted in the last 60 years to improve models which can be used instead of the classical ones [2].

In 1965, a new concept was developed by Lotfi A. Zadeh [3]. He developed the Fuzzy Set Theory which was concerned with uncertain and fuzzy data with the same importance as well-defined data. However, this concept started to be used only in the second half of the seventies, and this was regarding his articles in that period which were very effective and specific about the ability to apply fuzzy logic to systems that are characterized by uncertainty. In the second half of the eighties, fuzzy logic started to be used on some Japanese products which gave it a huge momentum and reached its current climax. Today, the applications of fuzzy logic can be observed in all fields. Additionally, the source is separated according to the application of the fuzzy logic, and each application is represented in a list according to the specification of its source. Instead of transferring the same list here, at this phase, it will be sufficient to list several fuzzy logic applications in the Electrical Engineering field [3]. The importance of the Fuzzy Set Theory is the ability to examine the real world and express it mathematically, so it goes beyond the boundaries of classical mathematics and involves uncertainty in the decision-making processes [4]. Many fields started to use this theory such as linear & nonlinear programming, operations research, goal programming, transportation models, and even in-game theory [5].

The first sign of Fuzzy PERT was in 1979 by Prade, and then, Chanas and Kamburowski discussed the Fuzzy PERT method and defined the times of activities by triangular fuzzy numbers [6]. They argued that the Beta distribution can be used only based on previous experiments and include 30 activities or more in the network [7]. Generally, there is no big difference between the classical way and the fuzzy one but when there are uncertain values, it became difficult to define a deadline to end a project; so, in this case, the methods of fuzzy logic trigonal should be conducted [8].

In this study, a Fuzzy PERT method which is a combination between the PERT model and the concept of Fuzziness was examined to be able to make decisions with uncertain values. In order to make it more understandable, this study will be conducted on a real-life project to build and establish a marble factory. The study includes the foundation of the construction process, the construction of the factory building, and the assembling of the machines that will be used in production. All these activities will be included, and the results will be examined.

2. Material and Methods

Lastly, it was observed clearly that the PERT & CPM techniques used in many studies were built on the Fuzzy concept. Day by day, the usage of these Fuzzy models is increasing due to the fantastic and effective innovation to solve the issue of uncertainty. The usage of Fuzzy Methods in almost all since and technology fields led to new perspectives in decision-making especially in industrial systems [9].. The literature studies show that the FPERT provides more realistic results than the classical model. With fuzzy models, the uncertainty of sources that will be planned in projects for the first time can be minimized or even eliminated by using fuzzy numbers.

Fuzzy Pert and Fuzzy CPM were both developed to use fuzzy numbers to determine the times of activities in the second half of the 1970s. The classical formula of Fuzzy CPM is to find the determinant activity times and replace them with fuzzy numbers. There are different definitions for the Critical Path which provide different estimations regarding the degree of criticality by using the same path in the system [10]Click or tap here to enter text.

In 1980, the Fuzzy shortest path and Fuzzy PERT/CPM were analyzed by Dubois and Prade [6]. To compare and add Fuzzy values they used extended addition/subtraction and fuzzy maximum/minimum. After that in 1981, Chanas and Kamburowski suggested an approach based on extended aggregation and strong level sets in solving fuzzy PERT [11]..

McCahon and Lee (1988) compared the comparison and composite method of Lee and Li (1987) to find and determine the project completion time, and they stated that the comparison between these methods is more understandable [12].

Based on probability theory, Buckley 1989 developed a Fuzzy PERT model. However, the early starts and slack times were still difficult to be calculated. On the other hand, Klein 1991 proposed an approach using a dynamic programming formulation to solve the Fuzzy shortcut problem.

In this study, a benchmarking method was used by Gencer and Türkbey in 2001, which is known as FPERT [13]. Gencer and Türkbey were assuming that the fuzzy time for each job is known in their projects. In the comparison method to find out the completion time of a project, the earliest fuzzy start-end in forwarding transition is defined as, and for backward transition, the latest fuzzy start-end is defined, and they should be calculated as shown below.

$$E\tilde{S}_i = \max_{v_j \in P_i} [E\tilde{S}_j (+) \tilde{A}_j] \quad (1)$$

$$E\tilde{F}_i = E\tilde{S}_i (+) \tilde{A}_i \quad (2)$$

$$L\tilde{F}_i = \min_{v_j \in S_i} [L\tilde{F}_j (-) \tilde{A}_j] \quad (3)$$

$$L\tilde{S}_i = L\tilde{F}_i (-) \tilde{A}_i \quad (4)$$

A_j, J is the fuzzy duration for a job, (+) fuzzy addition, (-) fuzzy subtraction, v_j, j. job, p_i, j is the premise set of work, S_i, j is the post-work set. While using triangular fuzzy numbers and in the cases when more than one job should be finished before another job can start, the premise fuzzy factors will be used to find the earliest start time of a new job $S(\tilde{A}_i)$, $m(\tilde{A}_i)$,

$m(\tilde{A}_i) > m(\tilde{A}_j)$ or $m(\tilde{A}_i) = m(\tilde{A}_j)$

and

$$s(\tilde{A}_i) < s(\tilde{A}_j) \quad (5)$$

When the conditions are fulfilled, $\tilde{A}_i > \tilde{A}_j$ will be accepted and, $\tilde{A}_i$ values will be considered as the earliest starting times. Here,

$m(\tilde{A}) = 1/3(a+b+c)$ and $s(\tilde{A})=1/18(a^2+b^2+c^2-ab-ac-bc)$ (6)

The above formulation will be accepted.

3. Results and Discussions

The activity periods which are considered triangular fuzzy numbers from companies A, B, and C are identified in Table 2. While calculating the average times, the taken time from each company has been estimated at a ratio of 0.33 and by summing all the time, different average values were founded.

The average value: $a_{(A,B,C)}, m_{(A,B,C)}, b_{(A,B,C)}$

$a_{(A,B,C)}$ = is taken as the average of good periods from A, B and C companies and it is calculated with the formula $a_{(A,B,C)} = a_A + a_B + a_C$,

while $a_{(A)} = A_1/3, = a_B/3, = C_1/3$

It is expressed as:

A_1 = the best activity period of company A, B_1 = the best activity period of company B, and C_1 = the best activity period of company C.

$m_{(A,B,C)}$ = is accepted as the average of the most probable periods taken from companies A, B, and C. the calculation method is the same method of calculating the best activity period.

$b_{(A,B,C)}$ = is accepted as the average of the best activity period taken from companies A, B, and C. the calculation method is the same method of calculating the best and the most probable periods of activity.

Table 1 is showing all the activities times, the premise activities, activities periods from 3 different companies, the average of the periods of these companies, the earliest starting time ES, the earliest ending time EF, the latest starting time LS, and the latest ending time LF [14]. According to the calculated times, the project completion time T is the job of O6, EF in the table. Below, this situation is expressed mathematically.

(x-93.6) / (126.1-93.6), 93.6 x 126.1

μ_t (x)= (156.6-x) / (156.6-126.1), 26.1 x 151.6

0 , 156.6 < x

It is calculated as Cpi = sup [Tpi ^ T], while Tpi in the formula is the fuzzy path length of i, ^ is the intersection of two sets, sup is the largest value of a set, and Tpi = $(+)_{j=pi}$ [13].

Table 1. The Activity Table

Code	Activity	Premise Activity	Activity Period	Average Value	ES	EF	LS	LF
	Excavation and Foundation Works							
A	Floor Arrangement	-	(1,2,5) (2,3,5) (3,4,6)	(2,3,5.3)	0	(2,3,5.3)	0	(2,3,5.3)
B	Drilling Column Pits	A	(2,2,5) (3,4,5) (2,4,5)	(2.3,3.3,5)	(2,3,5.3)	(4.3,6.3,10.3)	(2,3,5.3)	(4.3,6.3,10.3)
C	Laying Ground Sewer Pipes	B	(1,2,3) (2,2,4) (1,2,4)	(1.3,2,3.7)	(4.3,6.3,10.3)	(5.6,8.3,14)	(4.3,6.3,10.3)	(5.6,8.3,14)
D	Cracking Foundation Moulds in Column Pits	C	(3,4,6) (4,5,6) (3,5,7)	(3.3,4.7,6.3)	(5.6,8.3,14)	(8.9,13,20.3)	(5.6,8.3,14)	(8.9,13,20.3)
E	Laying Iron into Moulds	D	(5,6,7) (5,7,8) (6,7,9)	(5.3,6.7,8)	(8.9,13,20.3)	(14.2,19.7, 28.3)	(8.9,13,20.3)	(14.2,19.7,28.3)
F	Building Inspection	E	(0.4,0.5,0.6) (0.3,0.5,0.7) (0.5,0.7,0.9)	(0.4,0.6,0.7)	(14.2,19.7,28.3)	(14.6,20.3,29)	(14.2,19.7,28.3)	(14.6,20.3,29)
G	Pouring Concrete into Moulds and Hardening	F	(3,4,5) (2,4,6) (3,5,6)	(2.6,4.3,5.6)	(14.6,20.3,29)	(17.2,24.6,34.6)	(14.6,20.3,29)	(17.2,24.6,34.6)
H	Removal of Moulds	G	(1,2,2) (2,3,4) (1,2,3)	(1.3,2.3,3)	(17.2,24.6,34.6)	(18.5,26.9,37.6)	(17.2,24.6,34.6)	(18.5,26.9,37.6)
	Prefabricated Factory Building Construction Works							
G1	Placing Concrete Columns in Slots	H	(7,9,11) (8,9,10) (9,10,12)	(8,9.3,11)	(18.5,26.9,37.6)	(26.5,36.2,48.6)	(18.5,26.9,37.6)	(26.5,36.2,48.6)
G2	Connecting Columns with Beams	G1	(6,7,8) (5,6,9) (5,6,8)	(5.3,6.3,8.3)	(26.5,36.2,48.6)	(31.8,42.5,56.9)	(26.5,36.2,48.6)	(31.8,42.5,56.9)
G3	Pouring Concrete on the Factory Floor	G2	(3,4,5) (3,5,7) (4,6,7)	(3.3,5,6.3)	(31.8,42.5,59.6)	(35.1,47.5,63.2)	(31.8,42.5,56.9)	(35.1,47.5,63.2)
G4	Laying Concrete Floors on Administrative Building Floors	G3	(2,4,5) (3,4,5) (3,4,6)	(2.6,4,5.3)	(35.1,47.5,63.2)	(37.7,51.5,68.5)	(31.5,47.5,63.2)	(37.7,51.5,68.5)
G4’	Pouring Concrete at Assembly Points	G4	(1,2,2) (2,3,4) (2,3,3)	(1.6,2.6,3)	(37.7,51.5,68.5)	(39.3,54.1,71.5)	(37.7,51.1,68.5)	(39.3,54.1,71.5)
G5	Installation of Roof Panels	G4	(4,5,6) (4,6,7) (5,6,8)	(4.3,5.6,7)	(39.3,54.1,71.5)	(43.6,59.7,78.5)	(84.1,112.2,139. 4)	(88.4,117.8,146.4)
G6	Laying Rainwater Pipes	G5	(3,4,5) (2,3,3) (3,5,6)	(2.6,4,4.6)	(43.6,59.7,78.5)	(46.2,63.7,83.1)	(88.4,117.8,146. 4)	(91,121.8,151)
G7	Laying of Sewer Pipes	C	(3,4,5) (2,3,3) (3,5,6)	(2.6,4,4.6)	(5.6,8.3,14)	(8.2,12.3,18.6)	(49,65.7,85.6)	(51.6,69.7,90.2)
G8	Rainwater Line Laying	G6	(3,4,5) (2,4,6) (3,5,6)	(2.6,4.3,5.6)	(46.2,63.7,83.1)	(48.8,68,88.7)	(91,121.8,151)	(93.6,126.1,156.6)
G9	Welding Iron Profiles on Columns	G2	(16,19,21) (15,16,17) (18,20,21)	(16.3,18.3,19.6)	(31.8,42.5,56.9)	(48.1,60.8,76.5)	(43.4,66.5,89.1)	(59.7,84.8,108.7)
G10	Assembling of Wall Panels to Profiles	G9	(6,7,8) (5,6,9) (5,6,8)	(5.3,6.3,8.3)	(48.1,60.8,76.5)	(53.4,67.1,84.8)	(59.7,84.8,108.7)	(65,91.1,117)
G11	Installation of Factory Doors and Fire Doors	G10	(4,5,6) (3,4,4) (4,6,7)	(3.6,5,5.6)	(53.4,67.1,84.8)	(57,72.1,90.4)	(65,91.1,117)	(68.6,96.1,122.6)
G12	Laying of Electrical Installation in the Factory	G11	(21,24,26) (22,23,24) (20,25,28)	(21,24,26)	(57,72.1,90.4)	(78,96.1,116.4)	(68.6,96.1,122.6)	(89.6, 120.1,148.6)

G13	Laying of In-Factory Air Installation	G11	(2,2,5) (3,4,5) (2,4,5)	(2.3,3.3,5)	(57,72.1,90.4)	(59.3,75.4,95.4)	(91.3,122.8,151.6)	(93.6,126.1,156.6)
G14	Laying of In-Factory Plumbing	G11	(3,4,6) (4,5,6) (3,5,7)	(3.3,4.6,6.3)	(57,72.1,90.4)	(60.3,76.7,96.7)	(86.3,115.5,142.3)	(89.6,120.1,148.6)
G15	Production Units Partition with Sheet Metals	G11	(10,12,14) (9,11,12) (11,12,13)	(10,11.6,13)	(57,72.1,90.4)	(67,83.7,103.4)	(74.2,100.4,124.2)	(84.2,112,137.2)
	In-Factory Crane Installation							
H1	Laying Crane Rails with Mobile Crane	G15	(3,4,5) (2,3,3) (3,5,6)	(2.7,3.3,4.6)	(67,83.7,103.4)	(69.7,87,108)	(88.2,118.6,147)	(90.9,121.9,151.6)
H2	Sitting a Bridge on Two Rails	H1	(0.5,1,1.5) (0.4,0.6,1.2) (0.8,1,1.2)	(0.6,0.9,1.3)	(69.7,87,108)	(70.3,87.9,109.3)	(90.9,121.9,151.6)	(91.5,122.8,152.9)
H3	Assembling of Engines on the Bridge	H2	(0.4,0.5,0.6) (0.3,0.5,0.6) (0.5,0.7,0.8)	(0.4,0.6,0.7)	(70.3,87.9,109.3)	(70.7,88.5,110)	(91.5,122.8,152.9)	(91.9,123.4,153.6)
H3’	Laying of Electrical Cable Trays	G12, G14	(1,1.5,2) (0.9,1.3,1.6) (0.5,1,1.5)	(0.8,1.3,1.7)	(78,96.1,116.4)	(78.8,97.4,117.7)	(91.1,122.1,152.3)	(91.9,123.4,153.6)
H4	Pulling and Connection of Crane Electrical Installations	H3, H3’	(1,2,2) (2,3,4) (2,3,3)	(1.7,2.7,3)	(78.8,97.4,117.7)	(80.5,100.1,120.7)	(91.9,123.4,153.6)	(93.6,126.1,156.6)
	Establishment of the Knuckle Machine							
I1	Cracking Machine Wall Mouldings	G15	(3,4,5) (2,3,3) (3,5,6)	(2.6,4,4.6)	(67,83.7,103.4)	(69.6,87.7,108)	(84.2,112,137.2)	(86.8,116,141.8)
I2	Knitting of Machine Wall Irons	I1	(1,2,2) (2,3,4) (2,3,3)	(1.6,2.3,3)	(69.6,87.7,108)	(71.2,90,111)	(86.8,116,141.8)	(88.4,118.3,144.8)
I3	Pouring Machine Wall Concrete	I2	(0.3,0.4,0.5) (0.2,0.3,0.3) (0.3,0.5,0.6)	(0.3,0.4,0.5)	(71.2,90,111)	(71.5,90.4,111.5)	(88.4,118.3,144.8)	(88.7,118.7,145.3)
I4	Removal of Moulds	I3	(0.2,0.3,0.4) (0.1,0.2,0.2) (0.3,0.5,0.6)	(0.2,0.3,0.4)	(71.5,90.4,111.5)	(71.7,90.7,111.9)	(88.7,118.7,145.3)	(88.9,119,145.7)
I5	Placing the Machine Engine on the Concrete Floor	I4	(0.5,1,1.5) (0.4,0.6,1.2) (0.8,1,1.2)	(0.4,0.9,1.3)	(71.7,90.7,111.9)	(72.1,91.6,113.2)	(88.9,119,145.7)	(89.3,119.9,147)
I6	Body Assembly of Cutter Wires	I5	(1,2,5) (2,3,5) (3,4,6)	(2,3,5.3)	(72.1,91.6,113.2)	(74.1,94.6,118.5)	(89.3,119.9,147)	(91.3,122.9,152.3)
I7	Installation of Transfer Arm Between Motor and Cutter	I6	(0.5,1,1.5) (0.4,0.6,1.2) (0.8,1,1.2)	(0.6,0.5,1.3)	(74.1,94.6,118.5)	(74.7,95.1,119.8)	(91.3,122.9,152.3)	(91.9,123.4,153.6)
I7’	Laying of Electrical Cable Trays	G12, G14	(2,2,5) (3,4,5) (2,4,5)	(2.3,3.3,5)	(78,96.1,116.4)	(80.3,99.4,121.4)	(89.6,120.1,148.6)	(91.9,123.4,153.6)
I8	Assembly and Electrical Connection of Machine Electrical Panel	I7, I7’	(1,2,2) (2,3,4) (2,3,3)	(1.7,2.7,3)	(80.3,99.4,121.4)	(82,102.1,124.4)	(91.9,123.4,153.6)	(93.6,126.1,156.6)
	Establishment of Plate Wiping Line							
K1	Moulding into the Ground	G15	(1,1.5,2) (0.5,1,1.5) (1,1.5,2)	(0.8,1.3,3)	(67,83.7,103.4)	(67.8,85,106.4)	(85.5,112.9,138.2)	(86.3,114.2,141.2)
K2	Throwing Concrete on the Ground in a Balanced Way	K1	(0.4,1.1,2) (0.3,1.3,1.6) (0.3,1,1.2)	(0.3,1.1,1.6)	(67.8,85,106.4)	(68.1,86.1,108)	(86.3,114.2,141.2)	(86.6,115.3,142.8)
K3	Positioning the Machine	K2	(0.3,0.4,0.5) (0.2,0.3,0.3) (0.3,0.5,0.6)	(0.3,0.4,0.5)	(68.1,86.1,108)	(68.4,86.5,108.5)	(86.6,115.3,142.8)	(86.9,115.7,143.3)
K4	Installation of Input and Output Conveyors	K3	(2,4,5) (3,4,5) (3,4,6)	(2.7,4,5.3)	(68.4,86.5,108.5)	(71.1,90.5,113.8)	(86.9,115.1,143.3)	(89.6,119.7,148.6)

K5	Installing the Plate Loading Robot	K4	(2,3,5) (3,4,5) (2,4,5)	(2.3,3.7,5)	(71.1,90.5,113.8)	(73.4,94.2,118.8)	(89.6,119.7,148.6)	(91.9,123.4,153.6)
K5’	Laying of Electrical Cable Trays	G12, G14	(2,3,4) (1.5,2,4) (2,4,6)	(1.8,3,4.7)	(78,96.1,116.4)	(79.8,99.1,121.1)	(90.1,120.4,148.9)	(91.9,123.4,153.6)
K6	Machinery Electrical Installation and Connection	K5, K5’	(1,2,2) (2,3,4) (2,3,3)	(1.7,2.7,3)	(79.8,99.1,121.1)	(81.5,101.8,124.1)	(91.9,123.4,153.6)	(93.6,126.1,156.6)
	Bridge Cutting Machine Installation							
L1	Positioning the Machine	G15	(1,1.5,2) (0.9,1.3,1.6) (0.5,1,1.5)	(0.8,1.3,1.7)	(67,83.7,103.4)	(67.8,85,105.1)	(89.6,120.3,149.3)	(90.4,121.6,151)
L2	Levelling the Machine with Feet	L1	(1.2,1.6,2) (1,1.3,1.7) (0.5,1,1.5)	(0.9,1.3,1.7)	(67.8,85,105.1)	(68.7,86.3,106.8)	(90.4,121.6,151)	(91.3,122.9,152.7)
L2’	Laying of Electrical Cable Trays	G12, G14	(1,2,2) (2,3,4) (2,3,3)	(1.7,2.7,3)	(78,96.1,116.4)	(79.7,98.8,119.4)	(89.6,120.2,149.7)	(91.3,122.9,152.7)
L3	Machinery Electrical Installation and Connection	L2, L2’	(1.6,2,2.5) (1.4,2.2,2.8) (1.5,2,2.4)	(1.5,2.1,2.6)	(79.7,98.8,119.4)	(81.2,100.9,122)	(91.3,122.9,152.7)	(92.8,125,155.3)
L4	Making Cut Calibrations	L3	(0.8,1,1.4) (1,1.3,1.5) (0.6,0.9,1.1)	(0.8,1.1,1.3)	(81.2,100.9,122)	(82,102,123.3)	(92.8,125,155.3)	(93.6,126.1,156.6)
	Administrative Section Ground Floor Construction Works							
M1	Building a Wall with Brick	G4’	(8,11,12) (6,8,10) (5,9,11)	(6.3,9.3,11)	(39.3,54.1,71.5)	(45.6,63.4,82.5)	(39.3,54.1,71.5)	(45.6,63.4,82.5)
M1’	Opening Pipe Channels to Walls	M1	(4,5,6) (3,5,6) (5,7,8)	(3.3,5.7,6.7)	(45.6,63.4,82.5)	(48.9,69.1,89.2)	(48.3,64,83.5)	(51.6,69.7,90.2)
M2	Installation of Windows	M1	(3,4,5) (2,3,3) (3,5,6)	(2.7,4,4.7)	(45.6,63.4,82.5)	(48.3,67.4,87.2)	(50.2,68.4,88.5)	(52.9,72.4,93.2)
M3	Electrical Installation	M1	(6,8,11) (7,9,10) (9,10,11)	(7.3,9,10.7)	(45.6,63.4,82.5)	(52.9,72.4,93.2)	(45.6,63.4,82.5)	(52.9,72.4,93.2)
M4	Plumbing and Heating System Installation	M1	(2,2,5) (3,4,5) (2,4,5)	(2.3,3.3,5)	(45.6,63.4,82.5)	(47.9,66.7,87.5)	(50.6,69.1,88.2)	(52.9,72.4,93.2)
M5	Laying of Wastewater Pipes	M1’, G7	(1,2,2) (2,3,4) (2,3,3)	(1.3,2.7,3)	(48.9,69.1,89.2)	(50.2,71.8,92.2)	(51.6,69.7,90.2)	(52.9,72.4,93.2)
M6	Screed Removal	M2, M3, M4, M5	(4,5,6) (3,4,4) (4,6,7)	(3.7,5,5.7)	(52.9,72.4,93.2)	(56.6,77.4,98.9)	(52.9,72.4,93.2)	(56.6,77.4,98.9)
M6’	Watering the Screed for Hardening	M6	(0.6,0.8,1) (0.5,0.8,1) (0.5,0.6,0.7)	(0.5,0.7,0.9)	(56.6,77.4,98.9)	(57.1,78.1,99.8)	(56.6,77.4,98.9)	(57.1,78.1,98.9)
M7	Plaster Works	M6’	(9,11,13) (8,10,11) (10,11,12)	(9,10.7,12)	(57.1,78.1,99.8)	(66.1,88.8,111.8)	(60.6,85,107.9)	(69.6,95.7,119.9)
M8	Ceramics, Tile Works	M7	(4,5,6) (4,6,7) (5,6,8)	(4.7,5.7,7)	(66.1,88.8,111.8)	(70.8,94.5,118.8)	(73.9,100.7,124.9)	(78.6,106.4,131.9)
M9	Paint Works	M8	(2,2,5) (3,4,5) (2,4,5)	(2.3,3.3,5)	(70.8,94.5,118.8)	(73.1,97.8,123.8)	(79,106.4,131.9)	(81.3,109.7,136.9)
M10	Natural Gas Installation and Honeycomb Installation	M9	(4,5,6) (4,6,7) (5,6,8)	(4.3,5.7,7)	(73.1,97.8,123.8)	(77.4,103.5,130.8)	(81.3,109.7,136.9)	(85.6,115.4,143.9)
M11	Installation of Parquet, Doors, Cabinets	M10	(4,5,6) (3,4,4) (4,6,7)	(3.7,5,5.7)	(77.4,103.5,130.8)	(81.1,108.5,136.5)	(86.2,116.1,145.2)	(89.9,121.1,150.9)
	Administrative Section 1st Floor							

	Construction Works							
N1	Building a Wall with Brick	G4'	(8,11,12) (6,8,10) (5,9,11)	(6.3,9.3,11)	(39.3,54.1,71.5)	(45.6,63.4,82.5)	(50.8,68.8,88.8)	(57.1,78.1,99.8)
N1'	Opening Pipe Channels to Walls	N1, M6'	(4,5,6) (3,5,6) (5,7,8)	(3.3,5.7,6.7)	(57.1,78.1,99.8)	(60.4,83.8,106.5)	(59.8,78.7,100.8)	(63.1,84.4,107.5)
N2	Installation of Windows	N1, M6'	(3,4,5) (2,3,3) (3,5,6)	(2.7,4,4.7)	(57.1,78.1,99.8)	(59.8,82.1,104.5)	(61.7,83.1,105.8)	(64.4,87.1,110.5)
N3	Electrical Installation	N1, M6'	(6,8,11) (7,9,10) (9,10,11)	(7.3,9,10.7)	(57.1,78.1,99.8)	(64.4,87.1,110.5)	(57.1,78.1,99.8)	(64.4,87.1,110.5)
N4	Plumbing and Heating System Installation	N1, M6'	(2,2,5) (3,4,5) (2,4,5)	(2.3,3.3,5)	(57.1,78.1,99.8)	(59.4,81.4,104.8)	(62.1,83.8,105.5)	(64.4,87.1,110.5)
N5	Laying of Wastewater Pipes	N1', G7	(1,2,2) (2,3,4) (2,3,3)	(1.3,2.7,3)	(60.4,83.8,106.5)	(61.7,86.5,109.5)	(63.1,84.4,107.5)	(64.4,87.1,110.5)
N6	Screed Removal	N2, N3, N4, N5	(4,5,6) (3,4,4) (4,6,7)	(3.7,5,5.7)	(64.4,87.1,110.5)	(68.1,92.1,116.2)	(64.4,87.1,110.5)	(68.1,92.1,116.2)
N6'	Watering the Screed for Hardening	N6	(0.6,0.8,1) (0.5,0.8,1) (0.5,0.6,0.7)	(0.5,0.7,0.9)	(68.1,92.1,116.2)	(68.6,92.8,117.1)	(69.1,95,119)	(69.6,95.7,119.9)
N7	Plaster Works	N6', M7	(9,11,13) (8,10,11) (10,11,12)	(9,10.7,12)	(68.6,92.8,117.1)	(77.6,103.5,129.1)	(69.6,95.7,119.9)	(78.6,106.4,131.9)
N8	Ceramics, Tile Works	N7, M8	(4,5,6) (4,6,7) (5,6,8)	(4.7,5.7,7)	(77.6,103.5,129.1)	(82.3,109.2,136.1)	(78.6,106.4,131.9)	(83.3,112.1,138.9)
N9	Paint Works	N8, M9	(2,2,5) (3,4,5) (2,4,5)	(2.3,3.3,5)	(82.3,109.2,136.1)	(84.6,112.5,141.1)	(83.3,112.1,138.9)	(85.6,115.4,143.9)
N10	Natural Gas Installation and Honeycomb Installation	N9, M10	(4,5,6) (4,6,7) (5,6,8)	(4.3,5.7,7)	(84.6,112.5,141.1)	(88.9,118.2,148.1)	(85.6,115.4,143.9)	(89.9,121.1,150.9)
N11	Installation of Parquet, Doors, Cabinets	N10, M11	(4,5,6) (3,4,4) (4,6,7)	(3.7,5,5.7)	(88.9,118.2,148.1)	(92.6,123.2,153.8)	(89.9,121.1,150.9)	(93.6,126.1,156.6)
	Exterior and Landscaping							
O1	Establishment of Scaffolding for Exterior Insulation	N6	(3,4,5) (2,3,3) (3,5,6)	(2.7,4,4.7)	(68.1,92.1,116.2)	(70.8,96.1,120.9)	(68.1,92.1,116.2)	(70.8,96.1,120.9)
O2	Laying of Insulation Materials	O1	(6,8,9) (6,7,8) (5,7,9)	(5.7,7.3,8.7)	(70.8,96.1,120.9)	(76.5,103.4,129.6)	(70.8,96.1,120.9)	(76.5,103.4,129.6)
O3	Exterior Plaster and Paint	O2	(2,3,3) (3,4,5) (3,4,4)	(2.7,3.7,4)	(76.5,103.4,129.6)	(79.2,107.1,133.6)	(76.5,103.4,129.6)	(79.2,107.1,133.6)
O4	Building the Garden Wall	O3	(7,9,11) (8,9,10) (9,10,12)	(8,9.3,11)	(79.2,107.1,133.6)	(87.2,116.4,144.6)	(79.2,107.1,133.6)	(87.2,116.4,144.6)
O5	Parquet Stone Laying	O4	(5,8,10) (5,7,8) (4,6,9)	(4.7,7,9)	(87.2,116.4,144.6)	(91.9,123.4,153.9)	(87.2,116.4,144.6)	(91.9,123.4,153.6)
O6	Green Field Works	O5	(1,2,2) (2,3,4) (2,3,3)	(1.7,2.7,3)	(91.9,123.4,153.6)	(93.6,126.1,156.6)	(91.9,123.4,153.6)	(93.6,126.1,156.6)

For the marble factory establishment, there are 16 alternative routes between the beginning and the end of the project. Since all activity durations are 3-sided fuzzy numbers, fuzzy path lengths are calculated as the sum of the fuzzy durations of all activities in the route. These routes are listed as follows.

1. Route: A-B-C-D-E-F-G-H-G1-G2-G3-G4-G4'-M1-M3-M6-M6'-N3-N6-O2-O3-O4-O5-O6: **(90.9,122.1,151.9)**

2. Route: A-B-C-G7-M5-M6-M6'M7-M8-M9-M10-M11-N11 : **(41.40,56.10,76.60)**

3. Route: A-B-C-D-E-F-G-H-G1-G2-G9-G10-G11-G12-L2'-L3 L4: **(82,102,123,30)**

4. Route: A-B-C-D-E-F-G-H-G1-G2-G3-G4-G4'-N1-N2-N6-N6'-N7-N8-N9-N10-N11 : **(76.5,103.5,130.5)**

5. Route: A-B-C-D-E-F-G-H-G1-G2-G9-G10-G11-G15-K1-K2-K3-K4-K5-K6: **(75.1,96.9,121.8)**

6. Route: A-B-C-D-E-F-G-H-G1-G2-G3-G4-G4'-M1-M1'-M5-M6-M6'-M7-M8-M9-M10-M11-N11: **(82.10,112.90,141.2)**

7. Route: A-B-C-D-E-F-G-H-G1-G2-G3-G4-G4'-M1-M4-M6-M6'-N1'-N5-N6-O1-O2-O3-O4-O5-O6: **(85.90,119.80,149.90)**

8. Route: A-B-C-D-E-F-G-H-G1-G2-G3-G4-G5-G6-G8: **(47.20, 65.40,85.7)**

9. Route: A-B-C-D-E-F-G-H-G1-G2-G9-G10-G11-G13: **(58, 73.1, 92.40)**

10. Route: A-B-C-D-E-F-G-H-G1-G2-G9-G10-G11-G14-H3'-H4: **(62.8, 80.7,101.4)**

11. Route: A-B-C-D-E-F-G-H-G1-G2-G9-G10-G11-G14-K5'-K5: **(64.4, 83.4, 106.4)**

12. Route: A-B-C-D-E-F-G-H-G1-G2-G9-G10-G11-G14-I7-I8: **(62.6,79.9,101)**

13. Route: A-B-C-D-E-F-G-H-G1-G2-G9-G10-G11-G15-L1-L2 L3-L4: **(72.7,92.2,113.7)**

14. Route: A-B-C-D-E-F-G-H-G1-G2-G9-G10-G11-G15-H1-H2-H3-H4: **(72.4,91.2,113)**

15. Route: A-B-C-D-E-F-G-H-G1-G2-G9-G10-G11-G15-I1-I2-I3-I4-I5-I6-I7-I8: **(76.4,97.8,122.8)**

16. Route: A-B-C-D-E-F-G-H-G1-G2-G3-G4-G4'-M1-M2-M6-M6'-N4-N6-01-02-03-04-05-06: **(84, 115.4, 144.9)**

After calculating the durations of all alternative paths in the network diagram, the degrees of the criticality of all paths can be calculated. The degrees of the criticality of all paths were calculated in Figure 1 and are shown in Table 2.

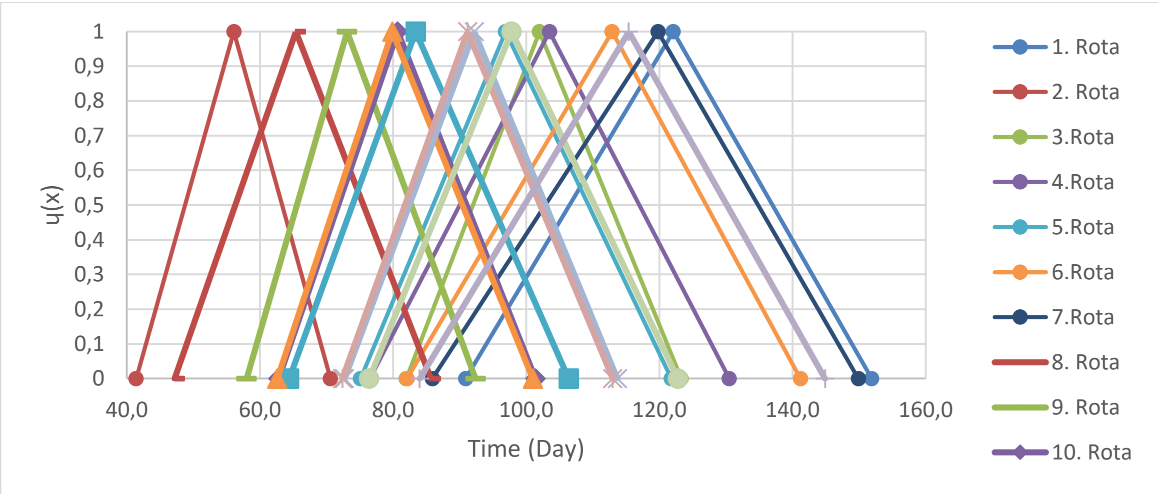

Figure 1. *Calculation of the Degrees of the Criticality of Alternative Routes* $(\boldsymbol{Cp_i})$

In this project management using the fuzzy PERT method, among the 16 alternative routes included in the whole network diagram, according to the results of the degrees of criticality in the chart, the 1st route as the critical path of the project, The route A-B-C-D-E-F-G-H-G1-G2-G3-G4-G4'-M1-M3-M6-M6'-N3-N6-O2-O3-O4-O5-06 was determined as (90.9,122.1,151.9) days as fuzzy completion times.

4. Conclusion

Project management is used extensively in production and the construction industry. Effective use of project management techniques is important because the construction process is difficult and involves many complex activities. The project management designed for the marble factory installation was implemented with the Fuzzy PERT method. As Chanas and Kamburowski discussed in the early eighties, this study converted the times of activities into triangular fuzzy numbers. Also, the huge number of activities and their complexity supported the possibility of applying the fuzzy concept effectively as mentioned in the literature review. Moreover, the study was very detailed and informative. It considered all the processes of

Table 2. *Degrees of the Criticality of the Paths*

Path	Cp_i
1. Route	1.00
2. Route	-0.44
3. Route	0.61
4. Route	0.68
5. Route	0.55
6. Route	0.84
7. Route	0.96
8. Route	-0.1
9. Route	0.03
10. Route	0.20
11. Route	0.28
12. Route	0.19
13. Route	0.43
14. Route	0.41
15. Route	0.56
16. Route	0.88

establishing and equipping a factory from the foundation works until the final accessory assembling bathing through building the production lines and setting up the machinery. Therefore, this application proves the efficiency of the used technique which is the FPERT method that had been developed by Gencer and Türkbey in 2001. According to the literature review, many studies relayed on fuzzy PERT in the last two decades to get more reliable results. This study should be an important addition to the other study because of the real data it worked on and the results it achieved. Furthermore, the study was very specialized in the marble industry, and this qualifies it to make great contributions either in the practical application or the research field in factories establishment generally or the marble industry particularly. It is expected to build on this article in different studies such as establishing factories in other sectors or in the same industry from different points of view, aspects, and details.

While a probability distribution is used in the estimation of the activity time in classical methods, there is no probability in the Fuzzy PERT technique and the working times are fuzzy, the project durations are calculated with uncertain limits. While making the calculations, unlike the classical PERT technique, the uncertainty dominates. This uncertainty can give the project team ideas about any deviations that may occur in the completion times of the projects. In other words, it provides flexibility to the project team while determining and planning the completion time of the project.

Author Statements:

- **Ethical approval:** The conducted research is not related to either human or animal use.
- **Conflict of interest:** The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper

- **Acknowledgement:** The authors declare that they have nobody or no-company to acknowledge.

- **Author contributions:** The authors declare that they have equal right on this paper.

- **Funding information:** The authors declare that there is no funding to be acknowledged.

- **Data availability statement:** The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

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