International Journal of Computational and Experimental Science and ENgineering (IJCESEN)

Vol. 9-No.2 (2023) pp. 133-140

http://dergipark.org.tr/en/pub/ijcesen

ISSN: 2149-9144

Research Article

Dynamic Modelling of the Spring Attached Two-Link Planar Manipulator

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Article Info:

DOI: 10.22399/ijcesen.1307444
Received: 30 May 2023
Accepted: 10 June 2023

Keywords

Two-link planar manipulator
Dynamic modelling
Lagrangian
Torsion springs

Abstract:

Robot dynamics is necessary not just for simulation and control, but also for the analysis of robot motion planners and controllers. There are a number of dynamic modelling investigations of robots with the different approaches and these investigations mostly focus on the link flexibility, joint friction or actuator dynamics. In addition to that these studies present dynamic model without any attached linear or torsion springs. In this study, the dynamic modelling of a two-link rigid manipulator with attached torsion springs is presented using the Lagrangian approach. The Lagrangian approach is a variational method that relies on the kinetic and potential energy of the mechanism, making it well-suited for the analysis of the two-link planar manipulator considered in this study. Initially, the equations of motion for the two-link rigid mechanism is derived without any attached torsion springs. Subsequently, two torsion springs are attached to the joints of the mechanism, and the existing equations of motion are modified accordingly. The study also presents the kinetic, potential and total energies of the two link-manipulator, angular positions of the links and their velocities. By considering the dynamic modelling of the torsion spring attached two-link rigid manipulator, this study contributes to understanding and analysis of the two-link manipulators and the dynamic effects of the attached springs.

1. Introduction

The representative dynamic model is essential and required to design and control of the mechanisms; therefore, having accurate dynamic model is very significant. There are different mechanisms for the different tasks and robotic manipulator mechanism is one them. The robotic manipulators are widely used in the industrial manufacturing tasks due to their versatility and precision. Some of these tasks require high sensitivity, where even small deviations can have significant consequences. The robotic manipulators typically consist of joints and links, which can be either rigid or flexible. The flexible manipulators have some advantages over the rigid manipulators such as low energy consumption, small size, larger workspace or light weight. Hence, a number of studies and designs have been investigated for the flexible manipulators. On the other hand, there are not many studies on the dynamic modelling of the rigid manipulators because rigid manipulators are simpler to model due to their inherent stiffness and lack of deformations.

The following articles present some of the dynamic modelling investigations: Khairudin et. al [1] investigates the dynamic modelling and characterization of a two-link flexible robot manipulator. This study combines Euler and Lagrange approaches, considering structural damping, hub inertia and payload. Subudhi and Morris [2] present the dynamic modelling and control of a manipulator with multiple flexible links and flexible joints. This study uses a combined Euler-Lagrange formulation, which simplifies the control of the complex two-link flexible manipulator. In this study, the results demonstrate good tracking performance and stabilization of the links. Chen [3] investigates the dynamic model of a multi-link flexible robotic manipulator. This study proposes a linearized dynamic model for a planar flexible manipulator with an arbitrary number of flexible links. The study employs the Lagrangian approach with Euler-Bernoulli beams and presents numerical simulations. Subedi et. al. [4] presents a closed-form dynamic model for planar multi-link flexible manipulators. This study utilizes the Lagrangian formulation and discuss the robot's configuration with a payload. The study focuses on both static and dynamic modelling. Morris and Madani [5] investigate static and dynamic modelling for a two-link flexible robotic manipulator. This study separately examines the elastic and rigid motions of the two links and then combine them using the superposition principle. Nicosia et. al. [6] presents dynamic modelling and experimental validation of a two-link flexible robot. This article provides an exact dynamic model for the robotic arm and carries out simulations along with experimental validation. Mayeda et. al. [7] investigates the base parameters of dynamic models for parallel and perpendicular manipulators with only rotational joints. This study focuses on non-redundant parameters of the dynamic model for these manipulators. Thomas and Tesar [8] present dynamic modelling for serial manipulator arms. This article derives the dynamic model for a serial manipulator with a rigid link model. Mehrjooee [9] conducts a non-linear dynamic analysis of a flexible-link manipulator. The study investigates the possibility of chaos occurrence in a two-link flexible robot mechanism and provides experimental validation for verification. Gamarra-Rosado and Yuhara [10] explore dynamic modelling and simulation of a flexible robotic manipulator with flexible links and two revolute joints. This article employs the Newton-Euler formulation to derive the dynamic model. Vakil et. al [11] proposes a new method for dynamic modelling of a flexible-link rigid-joint manipulator. This article introduces a dynamic model in terms of independent generalized coordinates without Lagrange multipliers. The method is validated through simulation examples. De Luca and Sciliano [12] present an explicit dynamic model for a planar two-link lightweight flexible robot. This article introduces a complete and accurate dynamic model using Euler-Bernoulli beams with uniform density, a standard Lagrangian approach, and rotary joints. Lochan et al. [13] provides a survey on two-link flexible robots. The article discusses dynamical analysis, complexities, modelling methods, and various aspects of works related to two-link flexible manipulator. Hasting and Book [14] present a linear dynamic model for the flexile robotic manipulators. This article introduces a linear state-space dynamic model for a single-link flexible manipulator. Arteaga [15] investigates some properties for the dynamic model of the flexible robotic manipulators. This study presents a dynamic model of the flexible manipulators based on the Lagrange's equations and discuses several significant properties of the presented dynamic model. Springs are commonly used in various robot designs to provide compliance, damping, or force/torque sensing capabilities. When springs are present, their dynamics should be properly incorporated into the overall dynamic model of the robot to accurately capture their effects on the system's behavior. In this study, the dynamic model of the torsion springs attached two-link rigid planar manipulator is presented. The dynamic model is developed using the Lagrangian formulation approach, which is a variational method based on the kinetic and potential energy of the system. Additionally, this study presents the positions and velocities of two-link planar manipulator rigid links and the kinetic, potential and total energies of the two-link manipulator without any attached springs. By presenting the dynamic model, this study contributes to the understanding and analysis of the two-link rigid planar manipulator with attached torsion springs. This information can be valuable for the designing controllers, analyzing system stability, and optimizing the manipulators performance in various applications.

2. Design of the Torsion Spring-Attached Two-Link Planar Manipulator

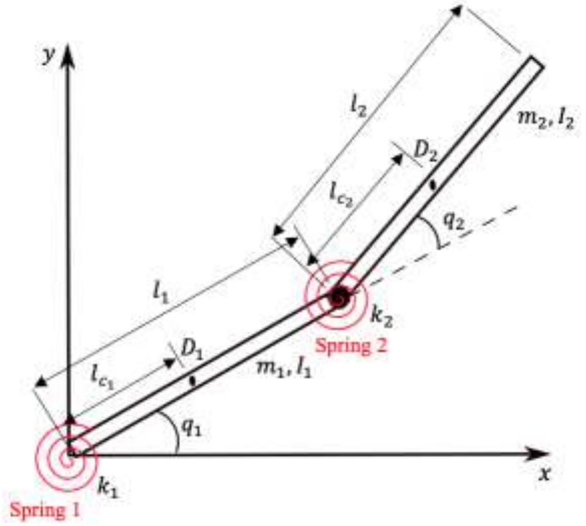

***Figure 1**. The two-link planar manipulator with two torsion springs.*

Figure 1 illustrates the configuration of a two-link planar rigid manipulator with two torsion springs and its corresponding parameters. The manipulator consists of two revolute joints with torsion springs attached, and the links are rigid. The parameters l_1 and l_2 represent the lengths of Links 1 and 2, respectively. The parameter m_1 corresponds to the mass of Link 1, while m_2 denotes the mass of Link 2. The parameter I_1 refers to the centroid moment of inertia for Link 1 and I_2 denotes the centroid moment of inertia for Link 2. The parameters τ_1 and τ_2 represent the joint torques for Joints 1 and 2, respectively. The parameters l_{c_1} and l_{c_2} indicate the center of mass positions of Links 1 and 2 relatives to

Joints 1 and 2, respectively. The parameter q_1 represents the angle of Link 1 relative to the x- axis, and the parameter q_2 represents the angle of Link 2 relative to Link 1. The parameters D_1 and D_2 represent the center of mass points of the Links 1 and 2 respectively. The parameter k_1 denotes the stiffness of Spring 1, and k_2 refers to the stiffness of Spring 2. It is important to note that the two-link planar rigid manipulator does not have any motion limitations.

3. Equations of Motion for the Two-Link Planar Manipulator

There are four primary parameters involved in dynamic modelling: the joint angle, q; the joint velocity, $\dot{q}$; the joint torque, τ; and the joint acceleration, $\ddot{q}$. Dynamic modelling formulations can be approached in two ways: forward dynamics and inverse dynamics. Forward dynamics involves providing the values of q, $\dot{q}$ and τ and investigating the parameter $\ddot{q}$. This approach is particularly useful for simulation purposes, as it allows for the prediction of joint accelerations and the resulting motion of the system. On the other hand, inverse dynamics involves providing the values of q, $\dot{q}$ and $\ddot{q}$, and investigating parameter τ. Inverse dynamics is commonly employed for robot control, as it enables the determination of the required joint torques to achieve a desired motion.

In this study, the dynamic modelling of the mechanism is initially presented without attached torsion springs. This step allows for a detailed presentation of the dynamic modelling process and minimizes potential formulation errors. Once the dynamic model of the mechanism is established, the torsion springs are then introduced and incorporated into the equations of motion accordingly. By introducing the torsion spring equations, the equations of motion for the two-link planar manipulator can be easily modified. This sequential approach ensures a comprehensive understanding of the dynamics modelling steps and facilitates the proper integration of the torsion springs into the system.As previously mentioned, the dynamic modelling in this study utilizes the Lagrangian approach. The Lagrangian method is an energy-based approach to dynamic modelling, defined by the following formula:

$$L(q,\dot{q}) = T(q,\dot{q}) - V(q) \tag{1}$$

where the parameter $L(q,\dot{q})$ denotes Lagrange, parameter $T(q,\dot{q})$ refers to the kinetic energy and the parameter $V(q)$ represents potential energy of the two-link manipulator. For a system with n-degree-of-freedom, the Euler-Lagrange formula is used to derive the equations of motion and is defined as follows:

$$\tau_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} \tag{2}$$

where the parameter τ_i represents the i^{th} joint torque, parameter L refers to the Lagrange function (Eq. (1)), parameter q_i refers to the angle of i^{th} joint, parameter $\dot{q}_i$ denotes to the i^{th} joint velocity, and parameter t represents time.

To further analyze the system, the kinetic energy formulas for Link 1 and Link 2 can be expressed separately. Starting with the kinetic energy formula for Link 1, it can be written as:

$$T_1 = \frac{1}{2}m_1 l_{c_1}(\dot{q}_1)^2 + \frac{1}{2}I_1(\dot{q}_1)^2 \tag{3}$$

where the parameter T_1 refers to the kinetic energy of the Link 1, the parameter m_1 represents the Link 1 mass, the parameter l_{c_1} denotes the center of mass of Link 1 from Joint 1, the parameter I_1 refers to the moment of inertia of the Link 1 and parameter $\dot{q}_1$ refers to the Joint 1 angular velocity.

The kinetic energy formula for Link 2 can be expressed as:

$$\begin{aligned} T_2 = \frac{1}{2}m_2 l_1^2(\dot{q}_1)^2 &+ \frac{1}{2}m_2 l_{c_2}(\dot{q}_1 + \dot{q}_2)^2 \\ &+ m_2 l_1 l_{c_2}\,\dot{q}_1 \cos q_2\,(\dot{q}_1 + \dot{q}_2) \\ &+ \frac{1}{2}I_2(\dot{q}_1 + \dot{q}_2)^2 \end{aligned} \tag{4}$$

where parameter T_2 represents the kinetic energy of Link 2, parameter m_2 is the mass of Link 2, parameter l_{c_2} denotes the distance from Joint 2 to the center of mass of Link 2, parameter I_2 is the Link 2 moment of inertia and the parameter $\dot{q}_2$ represents the angular velocity of Joint 2, parameter l_1 denotes the length of Link 1.

The total kinetic energy of the two-link planar manipulator can be defined as:

$$T(q,\dot{q}) = T_1(q,\dot{q}) + T_2(q,\dot{q}) \tag{5}$$

where parameter T represents total kinetic energy of the two-link planar manipulator. By substituting Eqns. (3) and (4) into Eq. (5), the following equation can be written:

$$\begin{aligned} T = \frac{1}{2}m_1 l_{c_1}(\dot{q}_1)^2 + \frac{1}{2}I_1(\dot{q}_1)^2 + \frac{1}{2}m_2 l_1^2(\dot{q}_1)^2 & \\ + \frac{1}{2}m_2 l_{c_2}(\dot{q}_1 + \dot{q}_2)^2 & \\ + m_2 l_1 l_{c_2}\,\dot{q}_1 \cos q_2\,(\dot{q}_1 + \dot{q}_2) & \\ + \frac{1}{2}I_2(\dot{q}_1 + \dot{q}_2)^2 & \end{aligned} \tag{6}$$

where the parameter T refers to the total kinetic energy of the two-link planar manipulator. As mentioned earlier, the potential energy of the mechanism is essential in the Lagrangian approach. The equation for the potential energy of Link 1 is defined as follows:

$$V_1 = m_1 g l_{c_1} \sin q_1 \tag{7}$$

where the parameter V_1 is the Link1 potential energy, the parameter g refers to the gravitational acceleration and the parameter q_1 represents the angle of Link 1.
The equation for the potential energy of Link 2 can be defined as:

$$V_2 = m_2 g\big(l_1 \sin q_1 + l_{c_2} \sin(q_1 + q_2)\big) \tag{8}$$

where parameter V_2 represents the Link 2 potential energy and parameter q_2 refers to the angle of Link 2 relative to Link 1. The total potential energy of the two-link planar manipulator is can be defined as:

$$V(q) = V_1(q) + V_2(q) \tag{9}$$

where the parameter V refers to the total potential energy of the two-link manipulator. By substituting Eqns. (7) and (8) into Eq. (9), the following equation can be obtained:

$$\begin{aligned} V = m_1 g l_{c_1} \sin q_1 & \\ + m_2 g\big(l_1 \sin q_1 & \\ + l_{c_2} \sin(q_1 + q_2)\big) & \end{aligned} \tag{10}$$

where the parameter V represents the total potential energy of the two-link manipulator. By substituting Eqns. (6) and (10) into the Lagrangian formula (Eq. (1)), the following equation can be written as:

$$\begin{aligned} L = \frac{1}{2} m_1 l_{c_1} (\dot{q}_1)^2 + \frac{1}{2} I_1 (\dot{q}_1)^2 + \frac{1}{2} m_2 l_1^2 (\dot{q}_1)^2 & \\ + \frac{1}{2} m_2 l_{c_2} (\dot{q}_1 + \dot{q}_2)^2 & \\ + m_2 l_1 l_{c_2}\, \dot{q}_1 \cos q_2\, (\dot{q}_1 & \\ + \dot{q}_2) + \frac{1}{2} I_2 (\dot{q}_1 + \dot{q}_2)^2 & \\ - m_1 g l_{c_1} \sin q_1 & \\ - m_2 g\big(l_1 \sin q_1 & \\ + l_{c_2} \sin(q_1 + q_2)\big) & \end{aligned} \tag{11}$$

where parameter L refers to the Lagrangian. The two-link rigid manipulator consists of two revolute joints, requiring the torque equations for these joints to be written separately. The Euler-Lagrange equations for Joint 1 and Joint 2 can be expressed as follows:

$$\tau_1 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) - \frac{\partial L}{\partial q_1} \tag{12}$$

$$\tau_2 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_2}\right) - \frac{\partial L}{\partial q_2} \tag{13}$$

where parameter τ_1 refers to the Joint 1 torque and parameter τ_2 represents to the Joint 2 torque. By differentiating Eq. (11) and substituting Eqns. (12) and (13), the resulting equations of motion can be expressed as:

$$\begin{aligned} \big[m_1 l_{c_1}^2 + m_2\big(l_1^2 + l_{c_2}^2 + 2 m_2 l_1 l_{c_2} \cos q_2\big) & \\ + I_{c_1} + I_{c_2}\big]\ddot{q}_1 & \\ + \big(m_2 l_{c_2}\big(l_1 \cos q_2 + l_{c_2}\big) & \\ + I_{c_2}\big)\ddot{q}_2 & \\ - \big(2 m_2 l_1 l_{c_2} \sin q_2\big)\dot{q}_1 \dot{q}_2 & \\ - \big(m_2 l_1 l_{c_2} \sin q_2\big)(\dot{q}_2)^2 & \\ + m_1 g l_{c_1} \cos q_1 & \\ + m_2 g\big(l_1 \cos q_1 & \\ + l_{c_2} \cos(q_1 + q_2)\big) = 0 & \end{aligned} \tag{14}$$

and

$$\begin{aligned} \big[m_2\big(l_{c_2}^2 + l_1\, l_{c_2} \cos q_2\big) + I_{c_2}\big]\ddot{q}_1 & \\ + \big[m_2 l_{c_2}^2 + I_{c_2}\big]\ddot{q}_2 & \\ + m_2 l_1 l_{c_2} (\dot{q}_1)^2 \sin q_2 & \\ + m_2 g l_{c_2} \cos(q_1 + q_2) = 0 & \end{aligned} \tag{15}$$

The Eqns. (14) and (15) represent the equations of motion for the planar two-link rigid manipulator without any attached springs.

4. Equations of Motion for the Torsion Springs Attached Two-Link Planar Manipulator

As mentioned earlier, the Lagrange equation without any attached springs is defined as:

$$L(q, \dot{q}) = T(q, \dot{q}) - V(q) \tag{16}$$

where the parameter $T(q, \dot{q})$ represents kinetic energy and $V(q)$ denotes potential energy of the mechanism. The torsion springs only possess potential energy. Therefore, the potential energies of the torsion springs need to be included in the potential energy component of the Lagrangian equation. As mentioned earlier, two torsion springs are attached to Joints 1 and 2. The potential energy of Spring 1 can be expressed as:

$$U_1 = \frac{1}{2} k_1 (q_1)^2 \tag{17}$$

where parameter U_1 refers to the Spring 1 potential energy, parameter k_1 represents the Spring 1

stiffness and parameter q_1 denotes the angle of Link 1.
Spring 2 potential energy can be expressed as:

$$U_2 = \frac{1}{2} k_2 (q_2)^2 \tag{18}$$

where the parameters U_2, k_2 and q_2 represent the Spring 2 potential energy, Spring 2 stiffness and angle of Link 2 respectively. The torsion springs are assumed massless in this study. According to the attached springs potential energies, the modified Lagrangian equation (Eq. (16)) can be written as follows:

$$L(q, \dot{q}) = T(q, \dot{q}) - [V(q) + U_1(q) + U_2(q)] \tag{19}$$

where parameters L, T and V represent Lagrange, kinetic energy and potential energy of the mechanism respectively and parameters U_1 and U_2 refer to in order to the Springs 1 and 2 potential energies. By substituting Eqns. (6), (10), (17) and (18) into the modified Lagrange equation (Eq. (19)), the resulting equation can be expressed as:

$$\begin{aligned} L = \frac{1}{2} m_1 l_{c_1} (\dot{q}_1)^2 + \frac{1}{2} I_1 (\dot{q}_1)^2 + \frac{1}{2} m_2 l_1^2 (\dot{q}_1)^2 \\ + \frac{1}{2} m_2 l_{c_2} (\dot{q}_1 + \dot{q}_2)^2 \\ + m_2 l_1 l_{c_2} \dot{q}_1 \cos q_2 (\dot{q}_1 \\ + \dot{q}_2) + \frac{1}{2} I_2 (\dot{q}_1 + \dot{q}_2)^2 \\ - m_1 g l_{c_1} \sin q_1 \\ - m_2 g \big(l_1 \sin q_1 \\ + l_{c_2} \sin(q_1 + q_2)\big) \\ - \frac{1}{2} k_1 (q_1)^2 - \frac{1}{2} k_2 (q_2)^2 \end{aligned} \tag{20}$$

For the Euler-Lagrange formulation of the mechanism, derivation of the torsion spring potential energy equations (Eqns. (17) and (18)) can be expressed as:

$$\frac{\partial U_1}{\partial q_1} = k_1 q_1 \tag{21}$$

$$\frac{\partial U_2}{\partial q_2} = k_2 q_2 \tag{22}$$

$$\frac{d}{dt}\left(\frac{\partial U_1}{\partial \dot{q}_1}\right) = \frac{d}{dt}\left(\frac{\partial U_2}{\partial \dot{q}_2}\right) = 0 \tag{23}$$

As observed in Eq. (23), the terms $\frac{d}{dt}\left(\frac{\partial U_1}{\partial \dot{q}_1}\right)$ and $\frac{d}{dt}\left(\frac{\partial U_2}{\partial \dot{q}_2}\right)$ will not affect the equations of motion as they are equal to zero. According to the Eqns. (21) and (22), the terms $\frac{\partial L_1}{\partial q_1}$ and $\frac{\partial L_2}{\partial q_2}$ will have terms $k_1 q_1$ and $k_2 q_2$ respectively. Since there is no change in the kinetic energy, the kinetic energy component of the equations of motion remains unmodified. By substituting Eqns. (21), (22) and (23) into the Euler-Lagrange equations (Eqns. (12) and (13)), the following modified equations of motion can be obtained:

$$\begin{aligned} \big[m_1 l_{c_1}^2 + m_2 \big(l_1^2 + l_{c_2}^2 + 2 m_2 l_1 l_{c_2} \cos q_2\big) \\ + I_{c_1} + I_{c_2}\big]\ddot{q}_1 \\ + \big(m_2 l_{c_2} \big(l_1 \cos q_2 + l_{c_2}\big) \\ + I_{c_2}\big)\ddot{q}_2 \\ - \big(2 m_2 l_1 l_{c_2} \sin q_2\big)\dot{q}_1 \dot{q}_2 \\ - \big(m_2 l_1 l_{c_2} \sin q_2\big)(\dot{q}_2)^2 \\ + m_1 g l_{c_1} \cos q_1 \\ + m_2 g \big(l_1 \cos q_1 \\ + l_{c_2} \cos(q_1 + q_2)\big) - k_1 q_1 \\ = 0 \end{aligned} \tag{24}$$

and

$$\begin{aligned} \big[m_2 \big(l_{c_2}^2 + l_1\, l_{c_2} \cos q_2\big) + I_{c_2}\big]\ddot{q}_1 \\ + \big[m_2 l_{c_2}^2 + I_{c_2}\big]\ddot{q}_2 \\ + m_2 l_1 l_{c_2} (\dot{q}_1)^2 \sin q_2 \\ + m_2 g l_{c_2} \cos(q_1 + q_2) \\ - k_2 q_2 = 0 \end{aligned} \tag{25}$$

The Eqns. (24) and (25) represent the modified equations of motion for the two-link planar robotic manipulator when two torsion springs are attached. As obtained in Eqns. (24) and (25), the inclusion of torsion springs introduces additional terms related to the stiffness and deformation of the springs, which affect the overall dynamics of the system.
The one of the important points about torsion spring is that they can be initially twisted, meaning they can have stored potential energy from the beginning. In such cases, the potential energy equations of the torsion springs will require an additional term for the initial twist angles, and the following equation can be written:

$$U_i = \frac{1}{2} k_i (q_i - q_{0_i})^2 \tag{26}$$

where the parameters U_i, k_i, q_i and q_{0_i} represent spring potential energy, spring constant, spring angle and spring initial angle, respectively. In this study, q_{0_i} assumed to be 0; thus, Eqns. (24) and (25) can be written.

5. Results

In this study, thee motion of two-link rigid planar manipulator is simulated using MATLAB. The initial conditions for the simulation are set as $q_1(0) = \pi/3$, $q_2(0) = \pi/4$, $\dot{q}_1(0) = 0$, $\dot{q}_2(0) = 0$ and the simulation time is set to 10 seconds. Throughout the 10-second simulation, a total of 921 data points for the position, velocity, potential energy, kinetic energy and total energy of the two-link rigid planar manipulator are recorded using MATLAB.

The parameters used in the simulation are assumed as follows: $l_{c_1} = 0.5l_1$ and $l_{c_2} = 0.5l_2$, where all other parameters are set to 1.0 in their respective consistent MKS units. The gravitational acceleration is assumed to be 9.8 m/s^2, and the mechanism is considered to have free oscillation. The torsion springs are assumed to be massless, and joint frictions, external forces and vibrations are neglected in the simulation. Note that this study specifically simulates the two-link planar manipulator without any attached-torsion springs. For the direction convention, the clockwise direction is considered positive, while the counter-clockwise direction is considered negative.

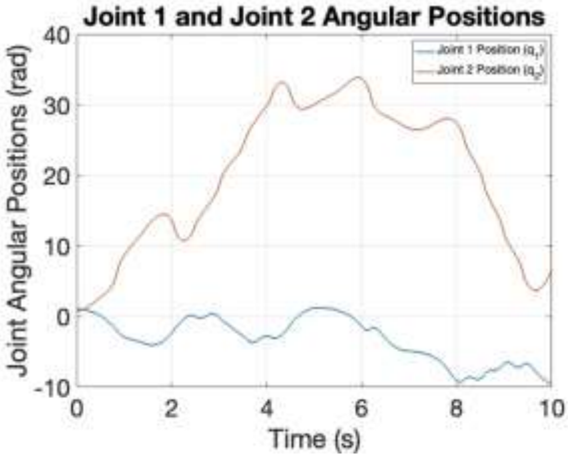

Figure 2. *The angular positions of the Link 1 and Link 2.*

Figure 2 illustrates the positions of Link 1 and Link 2, represented by the angles q_1 and q_2, respectively. Both of links undergo multiple rotations of 360 degrees during the 10-second simulation. The links sometimes change their rotation direction before completing a full 360 degrees. In the simulation, Link 1 rotates in the clockwise direction (assumed as the negative sign) from the beginning, while Link 2 rotates in the counter-clockwise direction (assumed as the positive sign). Consequently, the angle values of q_1 are negative, while the angle values of q_2 are positive. From Fig. 2, it can be observed that the initial values are $q_1 = 1{,}0472$ rad and $q_2 = 0{,}7854$ rad. After 10-second simulation, the final values are $q_1 = -9{,}8314$ rad and $q_2 = 6{,}6568$ rad. According to the Fig. 2, the Link 2 exhibits more pronounced oscillations compared to Link 1. It is important to note that the two-link manipulator mechanism is assumed to have free oscillation in this study. Figure 3 shows the velocities of Link1 and Link 2, represented by the angular velocities $\dot{q}_1$ and $\dot{q}_2$,

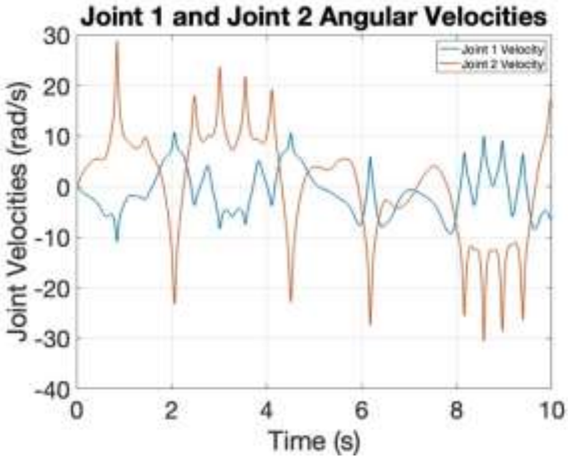

Figure 3*. The velocities of the Link 1 and Link 2.*

respectively. The velocities of both links exhibit positive and negative values due to the free oscillation of the mechanism. As a result, the links undergo rotations in both counter-clockwise and clockwise direction during the simulation, leading to changes velocity directions. From Fig. 3, it can be observed that the fluctuation of $\dot{q}_2$ is greater than the fluctuation of $\dot{q}_1$. Additionally, the rotation of link 2 is faster compared to that of Link 1. During the 10-second motion simulation, the values of $\dot{q}_2$ range between $-30{,}5479$ rad/s and $28{,}8705$ rad/s, while the values of $\dot{q}_1$ range between $-10{,}9336$ rad/s and $10{,}7907$ rad/s. as shown in Fig. 3.

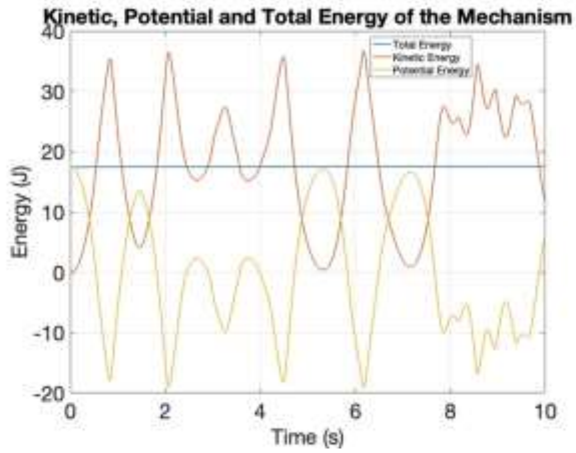

Figure 4. *The total, kinetic and potential energy of the two-link manipulator.*

Figure 4 illustrates the total, potential and kinetic energy of the two-link planar manipulator. As seen in Fig. 4, the total energy of the system remains constant throughout the motion simulation, with a value of $17{,}4636$ J. This constancy validates the accuracy of the dynamic modelling in this study, as friction and attached springs were not considered. The potential and kinetic energy exhibit time-varying behavior during the simulation, characterized by symmetric fluctuations. At the beginning of the motion simulation, the kinetic energy is zero since the mechanism is initially at rest. Conversely, the potential energy starts with a value of $17{,}4636$ J, equal to the total energy of the system. As shown in Fig. 4, some potential energy values are negative, which occurs when certain positions of Link 1 and Link 2 fall below the reference x- axis. The kinetic energy values range from 0 to $36{,}7360$ J, while the potential energy values range from $-19{,}1888$ J to $17{,}4636$ J.

6. Discussion

In this article, the torsion springs attached two-link manipulator plots are not presented. The addition of torsion springs imposes limitations on the motions of the manipulator, resulting in different joint positions and velocities compared to the plots shown in this article. Furthermore, due to the potential energy stored in the torsion springs during motion, the total energy of the spring-attached two-link manipulator will not remain constant. The plots and comparison of the spring-attached two-link manipulator with case of no attached springs will be one of the future investigation directions. This will provide valuable insights into the impact of torsion springs on the dynamics of the manipulator and further enhance our understanding of the system's behavior. According to the results, it is observed that the rotation of Link 2 is faster compared to Link 1. This discrepancy can be attributed to the configuration of the mechanism, where Link 1 is fixed to the ground while Link 2 is connected to Link1. Due to the free oscillation of the system, Link 2 undergoes more rotations than Link 1 throughout the 10-second motion simulation in this study. The torsion springs in the system do not possess any kinetic energy. As a result, the modification in the equations of motion is limited to potential energy component. Only two additional terms, namely $k_1 q_1$ and $k_2 q_2$, are introduced due to the attachment of the torsion springs. Consequently, the overall equations of motion do not undergo significant changes. The influence of the torsion springs is primarily manifested though these two terms, ensuring that the modifications to the equations of motion remain relatively minor.

7. Conclusion

In this study, dynamic modelling of a two-link rigid planar manipulator with attached torsion springs is investigated. Initially, the dynamic model of the two-link manipulator is presented without the torsion springs. Subsequently, the equations of motion are modified to incorporate the torsion springs attached to the joints of the manipulator. The addition of the torsion springs does not significantly alter the equations of motion. The results indicate that the total energy of the two-link manipulator remains constant, validating the accuracy of the dynamic model without the presence of attached springs. Furthermore, since the two-link manipulator exhibits free oscillation in this study, the position and velocity graphs illustrate the effects of this oscillatory behavior on the motion of the mechanism.

Author Statements:

- **Ethical approval:** The conducted research is not related to either human or animal use.
- **Conflict of interest:** The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper
- **Acknowledgement:** Author wishes to express his gratitude to the Ministry of National Education of the Republic of Türkiye which indirectly made this work possible.
- **Author contributions:** The authors declare that they have equal right on this paper.
- **Funding information:** The authors declare that there is no funding to be acknowledged.
- **Data availability statement:** The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

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