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Research Article

Construction of an Edge Detection Mask based on Caputo Fractional Derivative

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Abstract:

Fractional calculus has recently attracted significant attention, making substantial contributions to the field of digital image processing. This growing interest stems from its ability to offer a more nuanced mathematical framework for analysing and processing images. In this paper, we present an innovative approach to edge detection that leverages the Caputo fractional derivative. Unlike traditional edge detection methods, which may overlook subtle variations in pixel intensity, our approach utilizes the Caputo definition to enhance the edge identification process, thereby capturing finer details in the image. By applying this fractional derivative, we achieve a more precise and detailed representation of edges, which is particularly useful in scenarios requiring high accuracy. The effectiveness of our method is quantitatively assessed using the Peak Signal to Noise Ratio (PSNR), a metric that measures the quality of the processed image relative to the original. The results indicate that the Caputo fractional derivative not only effectively highlights edges but also maintains an optimal balance between preserving intricate details and minimizing noise, making it a powerful tool in digital image processing.

1. Introduction

The field of digital image processing refers to processing digital images by means of a digital computer. A digital image is comprised of a finite number of elements, each of which has a particular location and value called pixels. The processes of acquiring an image of the area containing the text, preprocessing that image, extracting the individual characters, describing the characters in a form suitable for computer processing, and recognizing those individual characters are in the scope of what we call digital image processing.[1]
Edges, lines, and points carry a lot of information about the various regions in the image. The edge detection operation is essentially an operation to detect significant local changes in the intensity level in an image. An ideal edge detector is required to detect an edge point precisely in the sense that a true edge point in an image should not be missed, while a false, non-existent edge point should not be erroneously detected.[2] Usually, edge detection algorithms are based on integer order differentiation operators. The two primary types of gradient masks used for edge identification are first-order derivative masks and second-order derivative masks. First-order derivative operators, such as Sobel, Prewitt, or Roberts operators, are used to identify the peaks, and second-order derivative operators, such as the Laplacian operator, are used to identify the zero crossings on the edges.
Fractional calculus is an old and mysterious mathematical discipline dealing with the non-integer order differentiation and integration which is proposed in the seventeenth century and developed mainly in the nineteenth century [3]. Fractional calculus came into light when Oldham and Spanier collaborated and published a book [4] in 1974 particularly devoted to fractional calculus
The Fractional Calculus is commonly seen as a broader version of the ordinary calculus that handles non-integer integration and differentiation orders. According to the definitions of Caputo and Grunwald-Letnikov, the fractional order derivative can be computed using a variety of operators, and

these definitions are found to be easily applicable in digital image processing applications.

When first-order derivative operators are used instead of second-order ones, thicker borders are produced, but some of the image details are lost. To efficiently detect the fine features, second-order derivative methods should be employed, but at the cost of increased noise sensitivity. Recently, fractional order derivative masks have emerged as a key solution to resolve this conflict. Fractional order masks improve edge detection techniques significantly in terms of noise sensitivity. By focusing on particular frequency components within the image, they also assist in regulating certain aspects of the image.

To address image processing problems, several methods based on fractional order approaches have been developed.

In [5], a 1D digital fractional-order Charef differentiator is extended to 2D via a multi-directional edge detection operator, proving effective on texture images. Grey-scale edge detection with fractional adaptations shows improved performance; [6] evaluates six such algorithms on color images. A fractional gradient for quantized images is discussed in [7], including applications to astronomy. Prewitt-type filters using Caputo and Caputo-Fabrizio derivatives are proposed in [8], offering better visual and PSNR results. Fractional calculus in multispectral image fusion is analyzed in [9]. Enhanced masks using fractional derivatives and different directions are developed in [10]. A new Riemann-Liouville-based convolution mask is introduced in [11]. Image enhancement algorithms using the Riesz operator (FCD-1, FCD-2) are derived in [12]. Edge detectors based on Atangana–Baleanu integrals and various approximations, including Grunwald–Letnikov and Caputo-Fabrizio, are developed in [13] and [14].

In [15], a Caputo–Fabrizio-based operator is applied to medical images. Adaptive operators improving GL and Toufik–Atangana schemes are proposed in [16]. Riemann-Liouville-based enhancement and edge comparisons are made in [17], while [18] addresses 2D-DCT signal/image processing in MATLAB. Quaternion fractional derivatives for signal analysis are discussed in [19]. An improved Roberts method via fractional calculus is introduced in [20]. [21] examines edge detection for concrete wall cracks using classical methods. Outdoor haze images are degraded by noise, and existing methods often ignore the haze-noise interrelation, so [22] proposes a dual-branch dehazing-denoising architecture using dark channel prior, unsupervised networks, a mean sampler, and self-supervised learning, fused via convolutional neural network to achieve superior PSNR and SSIM results. Image enhancement using Caputo-Fabrizio derivatives and fractional order variation is proposed in [23], while [24] presents an improved fractional Canny method.

Two new GL-based masks are proposed with mathematical proofs in [25]. An adaptive enhancement algorithm using Otsu threshold and Markov Random Field is introduced in [26]. A directional-fractional method for medical image enhancement is presented in [27]. Applications in biomedical signal processing using fractional filters are explored in [28]. Fractional Fourier transform and hyperchaotic systems are used for optical image encryption in [29], while [30] proposes an image encryption method based on a novel fractional quantum logistic map. These are some of the literary works that use fractional calculus concepts for image processing tasks like edge detection.

In this paper, we present a new construction of fractional based convolution mask for image edge analysis using Caputo definition. This mask is incorporated into a procedure similar to the well-known Sobel method. We evaluate the performance of our new operator against other edge detection operators found in the literature using the standard criterion, which is based on conventional integer-order and fractional derivatives. The remainder of this paper is organized as follows: the basic definitions of fractional derivatives and the structure of fractional-masks are introduced in Section 2. In Section 3, we show how the proposed mask is constructed based on Grunwald-Letnikov integral and subsequently in Section 4 the performance of the proposed operator is discussed. In Section 5, the conclusions are elaborated.

2. Preliminary Results

Fractional masks offer certain advantages for edge detection in comparison to traditional integer-order masks. An image processing method for identifying an object's boundaries within an image is called edge detection. It detects changes in brightness discontinuities to function. In fields like image processing, computer vision, and machine vision, edge detection is utilised for data extraction and image segmentation.

Fractional masks bring additional flexibility and sensitivity to edge detection for several reasons like better handling of complex edges, fine control of sensitivity, noise robustness, texture preservation, adaptability to image content, enhanced feature detection and so on.

Roberts operator, Prewitt operator, and Sobel operator are the three most often utilised integer first-order derivative operators. These edge

detection filters are traditional gradient-based ones. On the basis of two convolution operators, they independently identify the horizontal and vertical edges.
The Prewitt operator is one of the most often used filters to identify the edges in an image. This method is predicated on using the central difference to approximate the first-order derivative. The following two kernels are used to convolve the image in order to produce the results:

$h_x =$

-1	0	1
-1	0	1
-1	0	1

$h_y =$

-1	-1	-1
0	0	0
1	1	1

The Sobel operator, which uses central finite differences as a basis, is another significant filter. The method's primary goal is to create more partnerships with pixels that are closer to the mask's centre than it is with the Prewitt operator. These are the convolution kernels that are employed in this technique:

$h_x =$

-1	0	1
-2	0	2
-1	0	1

$h_y =$

-1	-2	-1
0	0	0
1	2	1

$-\rho_0$	0	ρ_0
$-\rho_1$	0	ρ_1
$-\rho_2$	0	ρ_2

The Sobel operator will detect a large number of spurious edges with a coarse edge width, while being better than others and offering more precise edge direction information. The Sobel operator is more sensitive to diagonal edges than to horizontal and vertical edges, while the Prewitt operator is more sensitive to both kinds of edges. The integral differential operators of integer-order operators form the foundation of each of the kernels listed above. By considering the notions of fractional differential, several significant advances have been made in this field. Fractional differential operators have shown amazing results in recent years when used to enhance image quality, texture enhancement, noise reduction, and edge analysis.
A 3x3 fractional integral mask, also known as a fractional derivative mask, is a kind of filter or mask that is applied to an image in order to carry out fractional differentiation and integration operations. It is used in image processing and signal processing. The coefficients used to determine the fractional integral operation on an input image are represented as a matrix with fractional values. The picture and mask are convolved to create the output, which is a processed copy of the original input.
In this paper, we propose a novel approach to image processing using a 3x3 fractional integral mask. The fractional integral mask extends traditional integer-order operators to non-integer orders, enabling more refined and versatile image manipulation. We present the mathematical formulation of the mask, its properties, and demonstrate its effectiveness through various experimental results. The use of fractional masks in edge detection provides an advanced tool to capture complex image structures, adapt to diverse image content, and enhance the accuracy and robustness of edge detection algorithms.
In image processing, the following general form is one of the most crucial formulas for expanding fractional differential operators:

$$\frac{d^\alpha f(t)}{dt^\alpha} \approx f(t) + (-\alpha) f(t-1) + \frac{(-\alpha)(-\alpha+1)}{2!} f(t-2) + \frac{(-\alpha)(-\alpha+1)(-\alpha+2)}{3!} f(t-3) + \ldots \quad (1)$$

New fractional order masks will be created using the following structure in the remaining sections of the study.
As the initial primary structure, a 3 x 3 fractional integral mask is built according to [13]:

$h_x =$

$$h_y =$$

ρ_0	ρ_1	ρ_2
0	0	0
$-\rho_0$	$-\rho_1$	$-\rho_2$

Now we consider the Caputo fractional derivative of order α which is given by the equation

$$^{C}D^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}} \, d\tau, \; n-1 < \alpha < n \quad (3)$$

3. Proposed Methodology

Algorithm:

Step 1: Computing the coefficients ρ

The coefficients ρ that are used to build the masks for edge detection are calculated using the following formula by giving the fractional order α.

$$\rho_0 = 1$$
$$\rho_1 = \alpha$$

$$\rho_2 = \frac{\alpha(\alpha - 1)}{2}$$

$$\rho_3 = \frac{\alpha(\alpha - 1)(\alpha - 2)}{6}$$

These coefficients are derived from the Caputo fractional derivative definition. They determine the weights of the neighbouring pixels when computing the derivative.

Step 2: Creating masks h_x and h_y

The h_x and h_y convolution masks are constructed and utilised for identifying edges in the x and y directions, respectively.

$$h_x =$$

$??_0$	$??_1$	$??_2$
0	0	0
$-??_0$	$-??_1$	$-??_2$

$$h_y =$$

$-??_0$	0	$??_0$
$-??_1$	0	$??_1$
$-??_2$	0	$??_2$

Mask h_x is intended to identify vertical edges by stressing variations in the horizontal direction, while mask h_y is intended to identify horizontal edges by stressing variations in the vertical direction.

Step 3: Applying masks to the image

To obtain the edge responses, input a grayscale image and convolve it with the masks h_x and h_y. Convolution is used to apply each mask on the image and highlight the edges. By comparing pixel values in the horizontal and vertical directions, convolution with h_x will highlight vertical edges, and convolution with h_y will highlight horizontal edges.

Step 4: Computing edge magnitude

Compute the total edge magnitude using the formula by combining the edge responses from both directions.

$$edge_{magnitude} = \sqrt{edge_x^2 + edge_y^2}$$

This step uses the Pythagorean theorem to combine the edge responses from the x and y directions. The result is a single image where the intensity of each pixel represents the strength of the edge at that location.

Step 5: Calculation of PSNR

Using the original image and the edge-detected image as inputs, we measure the quality of the two images.

Mean Squared Error (MSE):

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} (original[i,j] - processed[i,j])^2$$

where m and n are the dimensions of the image.
The MSE measures the average squared difference between corresponding pixels in the original and processed images.

Peak signal to noise ratio (PSNR):

$$PSNR = 20\log_{10}\left(\frac{MAX_{PIXEL}}{\sqrt{MSE}}\right)$$

where MAX_{PIXEL} is the maximum possible pixel value of the image (e.g., 255 for 8-bit images). The PSNR is a logarithmic measure of the peak error between the original and processed images. A higher PSNR value indicates that the processed image is more similar to the original image, which is generally desirable for tasks like denoising or compression.
In the context of edge detection, the PSNR value helps to quantify how much the edge detection process has altered the original image.

4. Experimental Analysis

We present the experimental results in this section that were obtained in Python by using (2) on the test images, converting to gray scale and then applying the two fractional masks in horizontal and vertical directions by taking different values of fractional order α. The suggested fractional edge detectors are contrasted with traditional edge detection algorithms as Prewitt filters, Roberts, Canny, and Sobel. The Peak Signal to Noise Ratio (PSNR), as described by [22] and provided in the algorithm, has been used to evaluate the performance of the two suggested filters.
It is crucial to remember that when comparing statistic parameters, a higher PSNR value indicates a better statistical outcome.
The PSNR values on the final pictures produced by applying the suggested technique along with a few conventional edge detection algorithms are numerically analysed and presented in Tables 1 through 4. The suggested filters, h_x and h_y, are found to be more efficient with greater PSNR values, whereas the typical filters exhibit the lowest efficiency with lower PSNR values. We have considered the most popular images that are used in Image processing which are Lena, House and Monarch.
Figures 1 to 3, show a comparison of edge detection images obtained by using our proposed techniques and traditional methods. From Figures 1, 2 and 3: images of the left and right of first row are original image and the grayscale image, respectively. In the second row there are images processed by Sobel, Canny and Prewitt filters, respectively. Images of the third row are obtained by applying (2) with α values 0.25, 0.75 and 0.99, respectively. The proposed convolution masks give the best outcomes at α = 0.99 and the next best values at α = 0.75 and 0.25.

Figure 1. Comparison of different edge detection methods: (a) original image; (b) grayscale image, (c), (d) and (e) are obtained by using Sobel, Canny and Prewitt filters, respectively; (f), (g) and (h) are obtained by using (2) for α = 0.25, 0.75 and 0.99, respectively

Table 1. *Comparative study of filters in terms of PSNR for Lena image*

Sl. No.	Filter Type	PSNR (in dB)
1	Sobel filter	27.96
2	Canny filter	28.07
3	Prewitt filter	27.93
4	Proposed Mask with α = 0.25	54.71
5	Proposed Mask with α = 0.75	54.95
6	Proposed Mask with α = 0.99	55.05

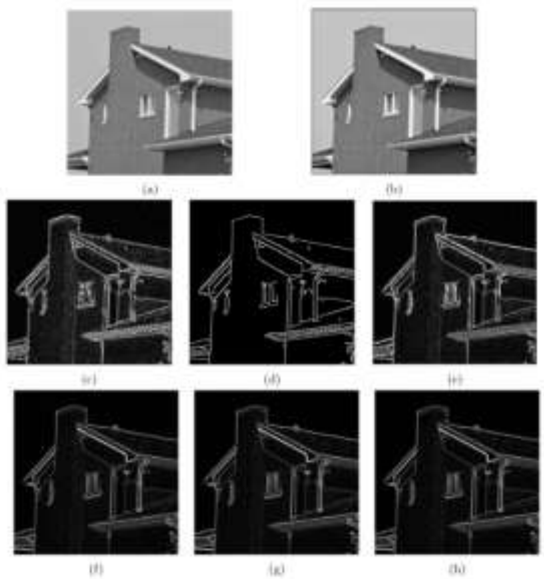

Figure 2. *Comparison of different edge detection methods: (a) original image; (b) grayscale image, (c), (d) and (e) are obtained by using Sobel, Canny and Prewitt filters, respectively; (f), (g) and (h) are obtained by using (2) for α = 0.25, 0.75 and 0.99, respectively.*

Table 2. *Comparative study of filters in terms of PSNR for House image.*

Sl. No.	Filter Type	PSNR (in dB)
1	Sobel filter	27.43
2	Canny filter	28.68
3	Prewitt filter	27.44
4	Proposed Mask with α = 0.25	53.73
5	Proposed Mask with α = 0.75	53.91
6	Proposed Mask with α = 0.99	54.00

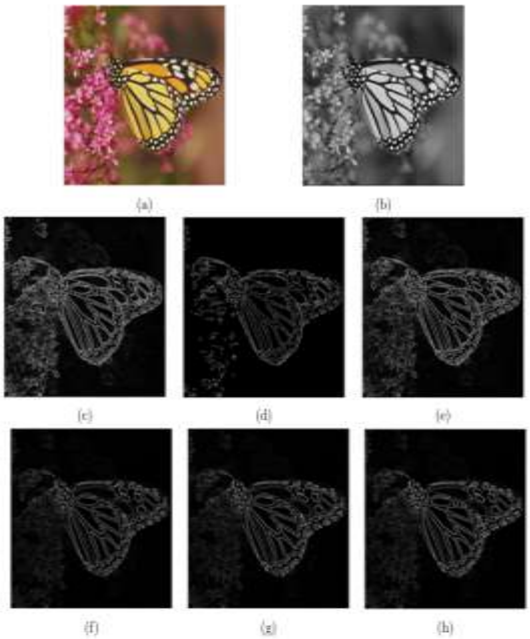

Figure 3. *Comparison of different edge detection methods: (a) original image; (b) grayscale image, (c), (d) and (e) are obtained by using Sobel, Canny and Prewitt filters, respectively; (f), (g) and (h) are obtained by using (2) for α = 0.25, 0.75 and 0.99, respectively.*

Table 3. *Comparative study of filters in terms of PSNR for Monarch image*

Sl. No.	Filter Type	PSNR (in dB)
1	Sobel filter	27.88
2	Canny filter	27.98
3	Prewitt filter	27.83
4	Proposed Mask with α = 0.25	53.72
5	Proposed Mask with α = 0.75	53.90
6	Proposed Mask with α = 0.99	53.99

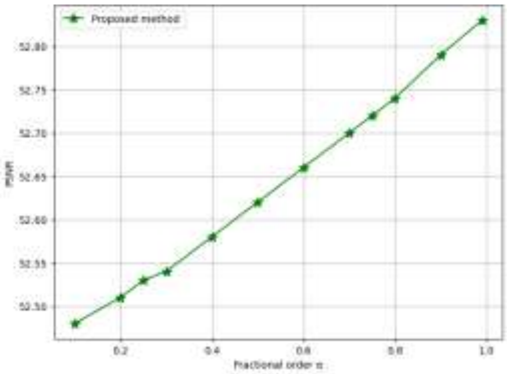

Figure 4. *PSNR trend with varying α using a* 3×3 *mask for Lena image*

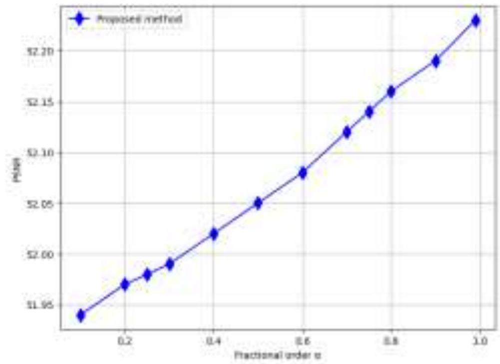

Figure 5. *PSNR trend with varying α using* 3×3 *mask for House image*

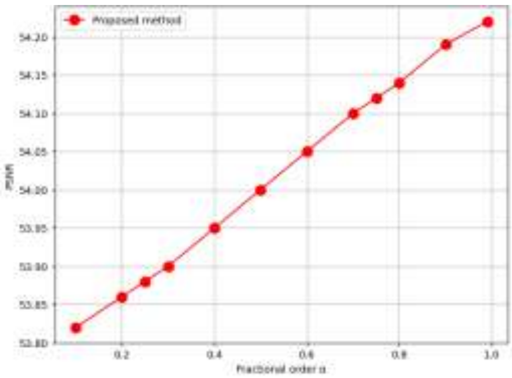

Figure 6. PSNR trend with varying α using a 3×3 mask for Monarch image

All three plots viz. Figure 4, Figure 5 and Figure 6 show a similar trend where PSNR increases with fractional order α, highlighting the effectiveness of the proposed method across the three different images. While the exact PSNR values differ slightly, reflecting the unique characteristics of each image, the overall performance pattern is consistent, with the Monarch image achieving the highest PSNR, followed by the Lena and House images. Overall, the proposed method enhances the PSNR values across all images, with performance varying slightly depending on the content and complexity of each image. The PSNR values show an increasing pattern when we adjust fractional orders with α because this method demonstrates enhanced sensitivity in image quality when nearing the value of 1.

5. Conclusion

This study introduces an effective edge detection technique based on the Caputo fractional derivative, providing a significant improvement over traditional edge detection methods. The suggested approach successfully improves edge features in grayscale images by determining certain coefficients from the fractional order values lying between 1 and 2. The algorithm's ability to detect edges is validated using the PSNR metric, which indicates a balanced performance between edge enhancement and noise suppression. The application of fractional calculus, specifically the Caputo definition, offers a flexible and robust framework for image processing tasks, particularly in scenarios where fine detail preservation is crucial. Traditional methods like Sobel, Canny, and Prewitt filters show moderate PSNR values around 27-28 dB, indicating effective edge detection with noticeable differences from the original image. The proposed masks with fractional derivatives achieve significantly higher PSNR values around 54 dB, indicating superior performance in preserving the original image's quality while detecting edges.
Our upcoming work will involve testing images with fractional order α values between 1 and 2, in view of that fact that second-order derivative methods have high capability in detecting fine details at the expense of their noise sensitivity. This work has focused on the 3x3 mask; however, in our perspective efforts, we would like to expand this to 5x5 and 7x7 masks.

Author Statements:

- **Ethical approval:** The conducted research is not related to either human or animal use.
- **Conflict of interest:** The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper
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