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Research Article

Exergetic Analyses of Hypersonic Flows

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Abstract:

Hypersonic devices and vehicles and hypersonic flows are very important and competitive advanced technology in science and in the World. Hypersonic technology is rapidly spreading and turning into competition in developed countries, especially in space studies and defense industry. This technology is vital for developed countries. Because of shock waves emerging in hypersonic devices during hypersonic flow, the energy and exergy destructions they cause are the most important difficulties, obstacles and problems which arise in this vital technology. To overcome these problems and difficulties, to prevent or reduce or minimize energy and exergy destruction, aerodynamic analysis and energy and exergy analysis methods are applied. The data and the results that can be guiding and useful for the calculations and the design have been obtained. In this study optimal design and optimum calculations ways of the energy and exergy destructions of hypersonic devices and instruments are discussed. It is obtained and shown that how much exergy destruction and entropy production occurs at which altitude and hypersonic velocities and also at which oblique shock angles.

1. Introduction

Flows above $5 < Ma$ are called hypersonic flows, flows between $1 < Ma < 5$ are called supersonic and flows less than $Ma < 1$ are called subsonic flows. Hypersonic devices and vehicles and hypersonic flows are very important advanced technology topics in space studies, in science, and defense industry. In developed countries this technology is rapidly turning into technological competition and becoming widespread. During hypersonic flow of a vehicle thin shock layers over the body of the vehicle occurs and called oblique shock. Besides these during hypersonic flow in the region of a shock detachment point an entropy layer occurs and then continues to flow downstream. Strong vorticities in this entropy layer that mostly cause analytical problems in the calculations and also, through the boundary layer viscous interaction of the mass flow is another important problem. Besides these problems extreme temperatures may occur in the hypersonic boundary layers and on the nose zone. The ionization might happen in the dissociation of the air molecules under these extreme temperatures [1, 2, 3].

Shock waves emerging in hypersonic flow and devices and the energy and exergy destructions they create are the most important obstacles and difficulties that arise in this technology. To reduce or to prevent or to minimize energy and exergy destruction in hypersonic vehicles lots of experiments have done in laboratories and in atmosphere. Some very important empiric equations and relations are found and applied to develop hypersonic devices.

Moorhouse [2, 3], and also Bejan [4, 5], have investigated and studied on this topic and proposed the second law of thermodynamics and exergy-based methods for analyses of the hypersonic vehicles. They obtained and showed that will allow better analyses and more complete systems integration.

In this study aerodynamic analysis, energy and exergy analyses methods are applied. Results and data that can be useful and guiding in the design have been obtained. Some optimal design methods of hypersonic instruments and devices are discussed and investigated. It is obtained and shown how much entropy production and exergy destruction occurs at which oblique shock angles and/or at which hypersonic velocities and altitudes.

2. Material and Methods

Hypersonic vehicles and flows are taken as materials in this study. As in figure 1 the optimal cruise scramjet hypersonic vehicles representation is given as examples for hypersonic vehicles. Energy analysis is done by using the equations;

$$\dot{Q}_{CV} - \dot{W}_{CV} + \sum_{in} \dot{m}_{in}\left(h_{in} + \frac{V_{in}^2}{2} + gz_{in}\right) - \sum_{out} \dot{m}_{out}\left(h_{out} + \frac{V_{out}^2}{2} + gz_{out}\right) = 0 \quad (1)$$

$\dot{W}_{CV}$ and $\dot{Q}_{CV}$ are the work and the heat energy, m is mass, h is enthalpy, z is altitude and v is speed. The law of conservation of mass in steady state is written as [6, 7].

$$\sum \dot{m}_{in} = \sum \dot{m}_{out} \quad (2)$$

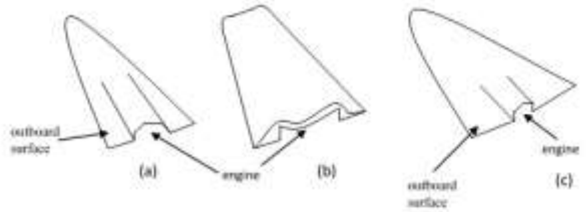

Figure 1. *The optimal cruise scramjet hypersonic vehicles schematic views.*

Exergy can be obtained if equilibrium with the environment is achieved at the end of a reversible process and it is the theoretical maximum amount of useful work. Chemical and physical exergy are the two components of it. For the perfect gas mixtures physical exergy for mixed substances can be written in molar terms as;

$$e_{phy} = (\bar{h} - \bar{h}_0)_{mix} - T_0.(s - s_0)_{mix} = \sum_i x_i\left[\int_{T_0}^{T} \bar{c}_{p0i}(T)dT - T_0.\left(\int_{T_0}^{T} \frac{\bar{c}_{p0i}(T)}{T}dT - \bar{R}\,ln\frac{P_i}{P_0}\right)\right] \quad (3)$$

In the equations, P is pressure, T is temperature, s is entropy, and x is molar rate. The maximum useful work which is achieved when a substance in the reference state becomes thermodynamic equilibrium in terms of chemical composition with its surroundings called chemical exergy [8, 9]. The chemical exergy of gas mixtures;

$$\bar{e}_{chem,mix} = \sum_i x_i.\bar{e}_{chem,i} + \bar{R}.T_0.\sum_i x_i.\,ln\,x_i \quad (4)$$

And, the total exergy of control mass or a flow is;

$$\bar{E} = \bar{E}_{phy} + \bar{E}_{chem} \quad (5)$$

Exergy equation for open systems is;

$$\sum_i \dot{m}_i h_i - \sum_i T_0 S_i - \sum_j \dot{m}_j h_j + \sum_j T_0 S_j + \sum \dot{Q}_k - \sum \dot{Q}_k \frac{T_0}{T_k} - \dot{W} = \dot{E}_{loss} \quad (6)$$

In Figure 2 oblique shock diagram is given, and in Figure 3 a) Shock angle versus deflection angle, and b) propulsion sub-system components are given. Mach number can be written as;

$$Ma = \frac{V}{c} = \frac{V}{\sqrt{kRT}} \quad (7)$$

The speed of sound is c;

$$Ma_{1n} = Ma_1.sin\beta \quad (8)$$

$$Ma_{1t} = Ma_1.cos\beta \quad (9)$$

$$Ma_{2n} = Ma_2.\sin(\beta - \alpha) \quad (10)$$

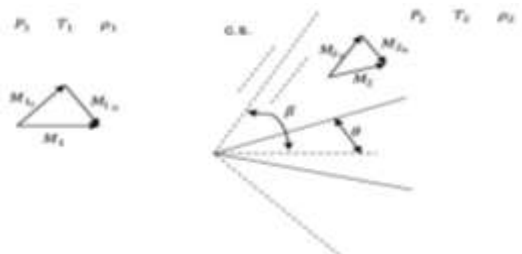

Figure 2. *Oblique shock diagram*

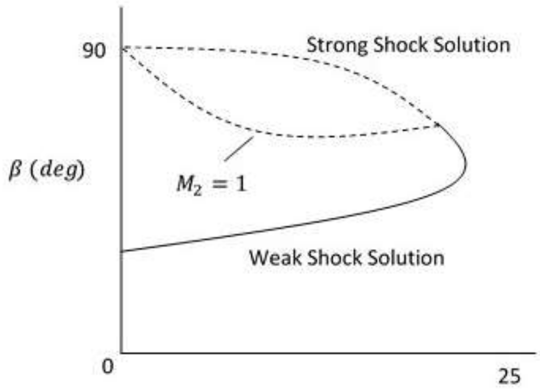

Figure 3. *Shock angle versus deflection angle*

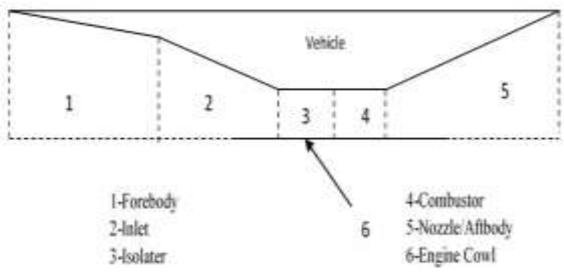

Figure 4. *Propulsion sub-system components.*

$$Ma_{2t} = Ma_2.\cos(\beta - \alpha) \quad (11)$$

$$tan\alpha = 2cot\beta.\frac{Ma_1^2.sin^2\beta - 1}{Ma_1^2.(k + cos^2 2\beta) + 2} \quad (12)$$

Since the angle α and Ma_1 are known, iteratively β strong shock angles and β weak shock angles can be calculated and other unknowns can be found by using this. The ratio of specific heats is k and k=1.4 is taken for air.

$$Ma_{2n} = \frac{Ma_{1n}^2 + 2/(k-1)}{Ma_{1n}^2 \cdot \frac{2.k}{k-1} - 1} \quad (13)$$

Pressure and temperature rates are;

$$\frac{P_2}{P_1} = \frac{k.Ma_{1n}^2 + 1}{k.Ma_{2n}^2 + 1} \quad (14)$$

$$\frac{T_2}{T_1} = \frac{(\frac{k-1}{2})Ma_{1n}^2 + 1}{(\frac{k-1}{2})Ma_{2n}^2 + 1} \quad (15)$$

The equation of the irreversible entropy is;

$$s_{irr} = s_2 - s_1 = c_p.\ln\left(\frac{T_2}{T_1}\right) - Rln\left(\frac{P_2}{P_1}\right) \quad (16)$$

The exergy destruction is;

$$I_{irr} = \dot{m}T_0\,(s_2 - s_1) \quad (17)$$

3. Results and Discussions

In Table 1, the results of the calculations for weak shock angle, P_2/P_1, Ma_1, T_2/T_1, Ma_2, ρ_1, ρ_2/ρ_1, $\dot{m}$, V, s_{irr}, and I_{irr} values for Ma_1=5 and for 2000 meters altitude, are given. Here the calculations are made for T1=250 K, Ma=5, and P1=1000 Pa. The altitude is defined as $H_{altitude}$= 2000 meters, and the speed of sound is taken for this altitude as c=332.5329 m/s, and ρ_1 is calculated as ρ_1 =1.00499 kg/m^3 for the Table 1 and Figure 4. As it can be seen that, increasing turn angle increases weak shock angles. Increasing turn angle also increases entropy and exergy destruction of the supersonic and hypersonic vehicles or systems. In Figure 5, variation of the entropy generation with turn angles in degrees (weak shock) for different Mach numbers for 2000 meters altitude are given. As can be seen that, increasing Mach number increases the entropy generation of the supersonic and hypersonic vehicles or the systems. For low Mach numbers these increases are low. However, for high Mach numbers these increases are very high. Also increasing turn angles in degrees (weak shock) increases the entropy generation. For low turn angles in degrees these increases are low. However, for high angles in degrees the entropy generation increases are very high. In Figure 6, variation of the entropy generation with turn angles in degrees (strong shock) for different Mach numbers for 2000 meters altitude are given. As can be seen that, increasing Mach number increases the entropy generation of the supersonic and hypersonic vehicles or the systems. For low Mach numbers these increases are low. However, for high Mach numbers these increases are very high. As can be seen that, increasing turn angles in degrees (strong

Table 1. *The calculation results of the weak shock angle, P_2/P_1, Ma_1, T_2/T_1, Ma_2, ρ_2/ρ_1, V, ρ_1, $\dot{m}$, s_{irr}, and I_{irr} values for 2000 meters altitude and for Ma_1=5 (V=1662.7 m/s, ρ_1=1.005 kg/m^3 and $\dot{m}$= ρ_1VA=1671 kg/s (A=1m^2)).*

Turn angle α^0	**weak shock angle β^0**	**Ma_2**	**P_2/P_1**	**T_2/T_1**	**ρ_2/ρ_1**	**$s_{irr}=s_2-s_1$ (kJ/kgK)**	**$I_{irr}=\dot{m}T_ç(s_2-s_1)$ (kW)**
2	12.85	4.79	1.277	1.0727	1.190	0.354	148
4	14.30	4.59	1.613	1.15	1.403	3.25	1356
6	15.87	4.39	2.016	1.234	1.634	10.1	4215
8	17.57	4.20	2.491	1.326	1.878	21.9	9165
10	19.38	3.99	3.044	1.429	2.13	39.3	16408
12	21.28	3.80	3.676	1.543	2.383	62.3	26014
14	23.29	3.60	4.392	1.67	2.632	90.7	37884
16	25.38	3.40	5.191	1.807	2.872	121.9	50947
18	27.55	3.21	6.073	1.958	3.101	157.6	65826
20	29.80	3.02	7.037	2.123	3.315	196.6	82130
22	32.13	2.84	8.084	2.299	3.515	237.2	99102
24	34.54	2.65	9.210	2.490	3.699	279.6	116809
26	37.04	2.48	10.41	2.693	3.868	323.1	134959
28	39.63	2.30	11.70	2.909	4.023	367.2	153404
30	42.34	2.13	13.07	3.138	4.164	400.1	167124
32	45.19	1.97	14.52	3.38	4.294	456.2	190559
34	48.24	1.80	16.06	3.639	4.413	501.3	209430

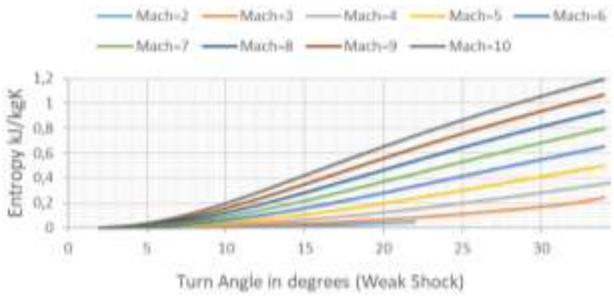

Figure 5. *Variation of the entropy generation with turn angles in degrees (weak shock) for different Mach numbers for 2000 meters altitude.*

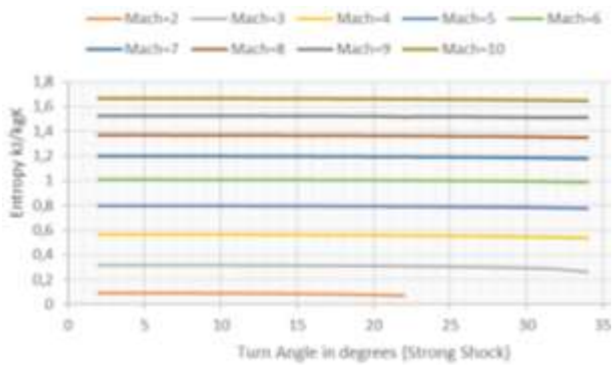

Figure 6. *Variation of the entropy generation with turn angles in degrees (strong shock) for different Mach numbers for 2000 meters altitude.*

shock) do not make any effect on the entropy generation which almost stay constant.

For the weak shock angle, the calculation results, in Table 2, for Ma_1, T_2/T_1, Ma_2, P_2/P_1, $\dot{m}$, ρ_2/ρ_1, ρ_1, V, s_{irr}, and I_{irr} values for Ma_1=5 altitude and for 24000 meters, are given. The calculations are made for P1=1000 Pa, Ma=5, and T1=250 K.

Table 2. *The calculation results for weak shock angle, Ma_2, P_2/P_1, T_2/T_1, ρ_2/ρ_1, $\dot{m}$, s_{irr}, and I_{irr} values for 24000 meters altitude (ρ_1=0.0412 kg/m^3, V=1488.92 m/s, $\dot{m}$= ρ_1VA=61.39 kg/s, (A=1m^2) and Ma_1=5).*

Turn angle α^0	weak shock angle β^0	Ma_2	P_2/P_1	T_2/T_1	ρ_2/ρ_1	$s_{irr}=s_2-s_1$ (kJ/kgK)	$I_{irr}=\dot{m}T_ç(s_2-s_1)$ (kW)
2	12.85	4.79	1.277	1.073	1.190	0.354	5.4
4	14.30	4.59	1.613	1.15	1.402	3.247	49.8
6	15.87	4.39	2.016	1.234	1.633	10.09	154.8
8	17.57	4.19	2.491	1.326	1.878	21.94	336.7
10	19.38	3.99	3.044	1.429	2.13	39.28	602.8
12	21.28	3.80	3.676	1.543	2.383	62.27	955.7
14	23.29	3.60	4.392	1.67	2.632	90.69	1391.8
16	25.38	3.40	5.191	1.807	2.872	121.96	1871.7
18	27.55	3.21	6.073	1.958	3.101	157.58	2418.4
20	29.80	3.02	7.037	2.123	3.315	196.61	3017.3
22	32.13	2.84	8.084	2.299	3.515	237.23	3640.9
24	34.54	2.65	9.210	2.490	3.699	279.62	4291.4
26	37.04	2.48	10.416	2.693	3.868	323.07	4958.3
28	39.63	2.30	11.701	2.909	4.023	367.22	5635.9
30	42.34	2.13	13.067	3.138	4.164	400.07	6140.0
32	45.19	1.97	14.516	3.38	4.294	456.17	7001.0
34	48.24	1.80	16.061	3.639	4.413	501.34	7694.3

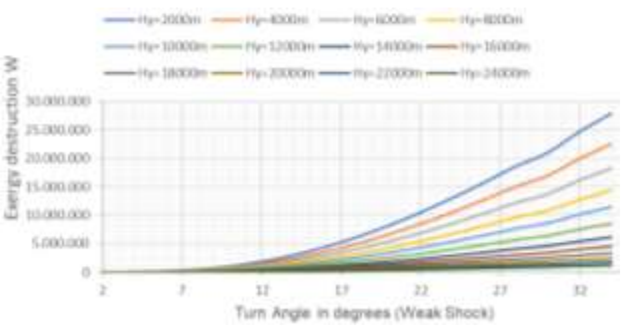

Figure 7. *Variation of the exergy destruction with turn angles in degrees (weak shock) for Ma=5 and different altitudes.*

The altitude is taken as $H_{altitude}$= 24000 meters and the speed of sound is taken for this altitude as c=297.784 m/sec, and ρ_1 is calculated as ρ_1=0.04123 kg/m^3, for the Figure 7 and Table 2. It can be seen that, increasing in turn angle values increases weak shock angles and that also means increasing turn angle also increases entropy and exergy destruction of the supersonic and hypersonic vehicles or systems.

In Figure 7, variation of the exergy destruction with turn angles in degrees (weak shock) for Ma=5 and different altitudes are given. As can be seen that, increasing altitudes decreases the exergy destruction of the supersonic and hypersonic vehicles or the systems. For low altitudes these increases are very high. However, for high altitudes these increases are very low. Also increasing turn angles in degrees (weak shock) increases the exergy destruction. For low turn angles in degrees these increases are low. However, for high angles in degrees the exergy destruction increases are very high for low altitudes. The density of the air is very effective on the entropy generation and on the exergy destruction. In Figure 8, variation of the exergy destruction with turn angles in degrees (strong shock) for Ma=5 and different altitudes are given. As can be seen that, increasing altitudes decreases the exergy destruction of the supersonic and hypersonic vehicles or the systems. For low altitudes these increases are very high. However, for high altitudes these increases are very low. As can be seen that, increasing turn angles in degrees (strong shock) do not make any effect on the entropy generation and on the exergy destruction which almost stay constant.

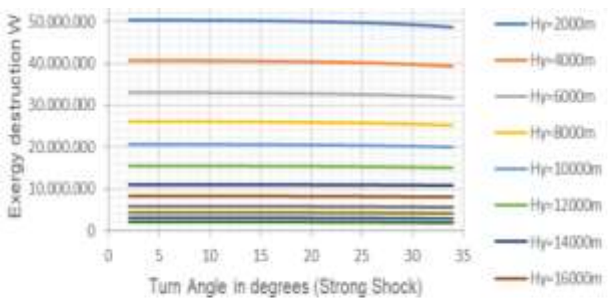

Figure 8. *Variation of the exergy destruction with turn angles in degrees (strong shock) for Ma=5 and different altitudes.*

It can be seen that; higher Mach means higher exergy destruction and higher altitudes means lower exergy destruction for the weak shock and the strong shock. Increasing turn angles in degrees for the strong shock do not make any effect on the entropy generation and on the exergy destruction which almost stay constant. Increasing turn angles in degrees for the weak shock makes rapidly increases on the entropy generation and on the exergy destruction. To decrease entropy generations of the supersonic and hypersonic vehicles or the systems, gradual compression method for the flow is very effective. In Figure 9, gradual compression of flow is given schematically. In Table 3 gradual compression results for 10^0 of turn angle are given. As can be seen that for 10^0 of turn angle the entropy generation is about 39 kJ/kgK. When 10^0 of turn angle is made as $10^0=5^0+5^0+0^0$ the entropy generation is about 13.3 kJ/kgK. When 10^0 of turn angle is made as $10^0=3^0+7^0+0^0$ the entropy

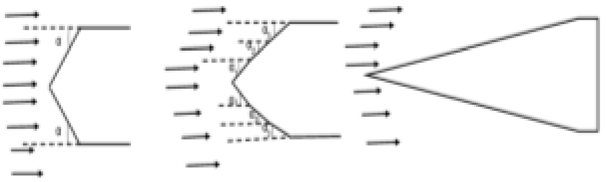

Figure 9. *Gradual compression of flow*

generation is about 17.8 kJ/kgK. The best solution $10^0=1^0+1^0+1^0+1^0+1^0+1^0+1^0+1^0+1^0+1^0$ seems to be **case8** as calculated its' total entropy 0.214 kJ/kgK.

Table 3. *Gradual compression results for 10^0 of turn angle.*

Case	$\alpha 1+\alpha 2+\cdots$	S_{irr} (kJ/kgK)
1	10	39.2793
2	3+7	17.8171
3	5+5	13.3394
4	4+6	13.3394
5	3+3+4	6.8489
6	2+4+4	6.8489
7	2+2+2+2+2	1.77
8	1+1+1+1+1+1+1+1+1+1	0.214

The designers of supersonic and hypersonic vehicles prefer empirically to gradually compression or contour surfaces. However, no matter how perfect the gradual compression or contour surfaces, oblique shock cannot be avoided. It is possible to provide some of its energy and exergy only by reducing the entropy production of the supersonic and hypersonic vehicles or the systems.

4. Conclusions

Hypersonic devices and vehicles and hypersonic flows are very important advanced technology which are rapidly turning into technological competition and becoming widespread in science and in the World. Hypersonic technology in developed countries, especially in space studies and defense industry, is vital for developed countries. In hypersonic flow and devices shock waves cause the energy and exergy destructions and they create the most important problems, obstacles and difficulties in calculations in this advanced technology which is very new.

In this study aerodynamic analysis and energy and exergy analysis methods have been applied. The results and data that can be guiding and useful in the design are investigated. For hypersonic devices and instruments optimal design ways of are discussed and investigated. It is concluded that higher Mach numbers causes higher exergy destruction and higher altitudes decreases the exergy destruction for the strong shock and the weak shock. For the strong shock increasing turn angles in degrees do not make any effect on the exergy destruction and on the entropy generation that almost stay constant. For the weak shock increasing turn angles in degrees causes rapidly increases on the exergy destruction and on the entropy generation. To decrease entropy generations of the supersonic and hypersonic vehicles or the systems, gradual compression method for the flow is very effective.

The designers of supersonic and hypersonic vehicles prefer empirically to gradually compression or contour surfaces. However, no matter how perfect the gradual compression or contour surfaces, oblique shock cannot be avoided. By reducing the entropy production of the supersonic and hypersonic systems or the vehicles some of its energy and exergy can be provided.

Author Statements:

- **Ethical approval:** The conducted research is not related to either human or animal use.
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- **Data availability statement:** The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

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