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Research Article

Riemann Integral versus Geometric Series in Natural Logarithmic Calculation: A Strategy to Enhance Student Competency for Sustainable Technology and Innovation

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Abstract:

The natural logarithm (ln) function plays a critical role in higher education, particularly in equipping students with advanced problem-solving skills applicable across science, technology, engineering, and mathematics (STEM) disciplines. This study has two primary objectives: first, to explore and compare the derivation of the natural logarithm using the Riemann integral and geometric series methods; and second, to examine the pedagogical implications of these approaches by analyzing student perceptions. Data were collected from 23 students using a questionnaire comprising five items, each scored on a scale of 1 to 10, to evaluate understanding and perception of both methods. Results from a paired t-test indicate that the Riemann integral method is considered superior to the geometric series in terms of conceptual understanding of the ln function ($p < 0.001$), ease of memorization ($p < 0.001$), manual computation ($p = 0.032$), and implementation in computer programming ($p < 0.001$). However, no significant difference was found between the two methods regarding the perceived difficulty of ln calculation ($p = 0.660$). Notably, the geometric series was favored for manual computations due to its simplicity.

1. Introduction

Education plays a crucial role in developing human potential to address societal challenges and achieve sustainability, with mathematics education fostering skills essential for technological innovation. Thanheiser identifies three frameworks, abstract knowledge and methods, contextual understanding, and human identity, that equip learners to solve real-world challenges [1-32]. Problem-solving, a critical competence, empowers students to formulate, solve, and interpret models across disciplines, supporting both technological advancements and sustainable practices. Embedding sustainability principles into problem-solving enables the development of innovative solutions to global challenges. Creative thinking in problem-solving depends on the educational context, the familiarity of applications, the availability of tools, and the reliability of evidence [4][31]. Researchers commonly employ tools such as open-ended questions, interviews, and the Torrance Test of Creative Thinking (TTCT). Previous studies highlight a positive correlation between mathematical thinking and attitudes, which are shaped by cognitive, affective, and behavioral components [6][15]. However, mathematical anxiety can negatively impact thinking and attitudes, ultimately interfering with performance [8][16]. To enhance conceptual understanding, educators are encouraged to integrate multiple representations of mathematical concepts [11].

The natural logarithm function, denoted as ln, is a fundamental concept in mathematics with significant applications in physics, economics, and engineering. Understanding this function provides essential tools for modeling exponential growth and decay, optimizing resources, and solving complex systems—critical skills for advancing sustainable technologies and innovations. For instance, the ln function is vital in developing energy-efficient systems, improving algorithms for renewable energy modeling, and analyzing environmental data for resource management. The natural logarithm can be defined in various equivalent ways, including as the inverse of the natural exponential function, an integral, a limit, and the unique continuous solution to a functional equation [21][27][29]. In engineering, particularly electrical and electronic engineering, logarithms are indispensable for interpreting and solving complex problems, optimizing solutions, and understanding mathematical relationships within computer programs [9][33]. Teaching efficient and accurate computational methods is therefore crucial, as it equips students with the necessary tools to address advanced mathematical and scientific challenges [26]. Mastery of the properties and applications of the ln function, along with the computational techniques involved, prepares students for a broad range of professional and research activities, making it a vital component of higher education curricula [24].

Two primary methods commonly taught in educational settings for calculating the ln function are the geometric series method and the Riemann integral method [2][5][18]. The geometric series method is based on the Taylor series expansion, offering a straightforward and efficient way to compute $\ln(1+x)$ within a certain range. This method is particularly effective for small values of x, where the series converges rapidly. Rapid convergence implies that only a few initial terms are needed to achieve a reasonably accurate approximation of the true value. However, this method is limited in its convergence range, as it is restricted to $|x| < 1$. For larger values of x, the series fails to converge, rendering this method ineffective. Understanding the Taylor series and its convergence properties is a vital part of mathematical education, given their frequent application in function analysis and differential equation solving.

In contrast, the Riemann integral method employs the integral definition to compute the natural logarithm function. Integration, a foundational concept in calculus, is used to calculate the area under a curve [7]. In this context, the integral of the function 1/t from 1 to x yields the value of ln(x). This approach can be approximated using numerical methods such as Riemann sums, the trapezoidal rule, and Simpson's rule. These techniques allow for the approximation of the integral by dividing the integration interval into smaller subintervals and summing the contributions from each. The Riemann integral method is more flexible, as it can be applied to all positive values of x and can achieve high accuracy using suitable numerical integration techniques. However, it typically involves more intensive computation and more complex algorithms compared to the geometric series method. Teaching the Riemann integral and related numerical techniques provides a solid foundation for understanding mathematical analysis and its computational applications.

In this study, we examine student perceptions of a comparative analysis between the geometric series method and a modified Riemann integral method for calculating the ln function. The modification to the Riemann integral method involves transforming the integral into a limit that more effectively approximates the integral. This limit offers an alternative definition of the natural logarithm via an integral-based approach. For instance, the value of $\ln(m)$ can be calculated using a limit that approximates the integral of 1/t over a specific interval. By applying appropriate Riemann sums, the integral value yielding ln(m) can be closely approximated. This approach highlights the flexibility of the Riemann integral method in various computational contexts and provides a deeper and more intuitive understanding of the natural logarithm function [18].

2. Materials and Methods

2.1. Study Design

This study aims to compare the computational efficiency and accuracy of the geometric series method and the modified Riemann integral method for calculating the natural logarithm ln function. The students in the class were of mixed abilities and were exposed to the same intervention. Identical instruments were used to collect data and analyze changes in their performance.

2.2. Participants

The participants in this study were 23 students enrolled in a Numerical Methods class in the Department of Mathematics at a public university in Indonesia. These students were in their fourth semester, during which numerical integration is a key component of the curriculum. The class was selected because it included a diverse group of students with varying levels of mathematical proficiency, providing a representative sample for the study.

2.3. Intervention

The intervention involved teaching students both the geometric series method and the modified Riemann integral method for calculating the ln function. Instruction was delivered through a combination of lectures, interactive computer-based tools, and practical exercises. The objective was to ensure that all students developed a thorough understanding of both methods and could apply them effectively.

2.4. The Riemann Integral Method

In this section, the proof of the Riemann integral method for deriving a formula for $\ln(m)$, where m is a natural number, is presented. The value of the definite integral

$$\int_a^b f(x)dx$$

is developed based on the Riemann sum:

$$\sum_{i=1}^{n} f(x_i)dx, \quad x_i=a+idx, \; i=1,2,3,\ldots,n, \quad dx=\frac{b-a}{n}$$

with n being a large number, or $n\to\infty$. The area bounded by the curve $y=1/x$, the horizontal axis $y=0$, and the vertical lines $x=a, x=b$ is given by the integral

$$L=\int_a^b \frac{1}{x}dx$$

Using the Riemann sum, this can be approximated as:

$$\sum_{i=1}^{n} \frac{1}{x_i}dx, \quad x_i=a+idx, \; i=1,2,3,\ldots,n.$$

It will now be proven that:

$$\ln(m)=\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{mn}\right), \quad m=2,3,4,\ldots.$$

Proof:
Define:

$$L_k=\int_k^{k+1}\frac{1}{x}dx, \quad k=1,2,3,\ldots.$$

We begin by using mathematical induction. For the base case, let k =1, i.e., m = 2. The exact solution is:

$$L_1=\int_1^2 \frac{1}{x}dx=\ln(2)-\ln(1)=\ln(2). \tag{1}$$

For the Riemann sum:

$$L_1=\sum_{i=1}^{n} f(x_i)dx=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{x_i}dx=\lim_{n\to\infty}\left(\frac{1}{1+1/n}\frac{1}{n}+\frac{1}{1+2/n}\frac{1}{n}+\frac{1}{1+3/n}\frac{1}{n}+\ldots+\frac{1}{1+n/n}\frac{1}{n}\right)=\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{n+n}\right)=\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{2n}\right). \tag{2}$$

By Equations (1) and (2),

$$\ln(2)=\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{2n}\right).$$

For m = 3, we proceed similarly. The exact solution is:

$$L_2=\int_2^3 \frac{1}{x}dx=\ln(3)-\ln(2). \tag{3}$$

For the Riemann sum,

$$L_2=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{x_i}dx=\lim_{n\to\infty}\left(\frac{1}{2+1/n}\frac{1}{n}+\frac{1}{2+2/n}\frac{1}{n}+\frac{1}{2+3/n}\frac{1}{n}+\ldots+\frac{1}{2+n/n}\frac{1}{n}\right)=\lim_{n\to\infty}\left(\frac{1}{2n+1}+\frac{1}{2n+2}+\frac{1}{2n+3}+\cdots+\frac{1}{2n+n}\right)=\lim_{n\to\infty}\left(\frac{1}{2n+1}+\frac{1}{2n+2}+\frac{1}{2n+3}+\cdots+\frac{1}{3n}\right). \tag{4}$$

From Equations (1), (3), and (4):

$$\ln(3)=\ln(3)-\ln(2)+\ln(2)=\ln(2)+L_2. \tag{5}$$

So,

$$\ln(3)=\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{2n}\right)+\lim_{n\to\infty}\left(\frac{1}{2n+1}+\frac{1}{2n+2}+\frac{1}{2n+3}+\cdots+\frac{1}{3n}\right)=\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{3n}\right). \tag{6}$$

Assume that the formula holds for m = k, with k ≥

2, $k \in Z$, so that:

$$\ln(k)=\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{kn}\right).$$

It will be shown that the statement also holds for $m = k + 1$. In other words, we will show that:

$$\ln(k+1)=\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{(k+1)n}\right). \quad (7)$$

Since

$$L_k=\int_k^{k+1}\frac{1}{x}\,dx=\ln(k+1)-\ln(k)$$

And

$$L_k=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{x_i}dx=\lim_{n\to\infty}\left(\frac{1}{k+1/n}\frac{1}{n}+\frac{1}{k+2/n}\frac{1}{n}+\frac{1}{k+3/n}\frac{1}{n}+\ldots+\frac{1}{k+n/n}\frac{1}{n}\right)=\lim_{n\to\infty}\left(\frac{1}{kn+1}+\frac{1}{kn+2}+\frac{1}{kn+3}+\cdots+\frac{1}{(k+1)n}\right). \quad (8)$$

Again, since

$$\ln(k+1)=\ln(k+1)-\ln(k)+\ln(k)=\ln(k)+L_k$$

we obtain:

$$\ln(k+1)=\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{kn}\right)+\lim_{n\to\infty}\left(\frac{1}{kn+1}+\frac{1}{kn+2}+\frac{1}{kn+3}+\cdots+\frac{1}{(k+1)n}\right)$$
$$=\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{(k+1)n}\right). \quad (9)$$

Therefore, it is proven that in general, the formula of $\ln(m)$ is:

$$\ln(m)=\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{mn}\right), m=2,3,4... \quad (10)$$

2.5. Geometric Series Method

The current numerical approximation of $\ln(x)$ is based on the expansion of an infinite geometric series. Salas et al. (Salas et al., 1986) explain that the value of $\ln(x)$ can be approximated using a formula derived from an equation where specific value within the interval $-1< x <1$ is implicitly solved on the left-hand side, while the right-hand side is obtained by substituting the solution.

$$\frac{1}{1-x}=1+x+x^2+x^3+x^4+x^5+\cdots, -1<x<1. \quad (11)$$

By integrating both sides of Equation (11), we obtain Equation (12):

$$-\ln(1-x)=x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{x^5}{5}+\frac{x^6}{6}+\cdots, -1<x<1. \quad (12)$$

Similarly, the infinite geometric series is determined for Equation (13):

$$\frac{1}{1+x}=1-x+x^2-x^3+x^4-x^5+\cdots, -1<x<1. \quad (13)$$

By integrating both sides of Equation (13), we obtain Equation (14):

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}-\frac{x^6}{6}+\cdots, -1<x<1. \quad (14)$$

Then, by adding Equations (12) and (14), we derive:

$$\ln(1+x)-\ln(1-x)=\ln\frac{1+x}{1-x}=2\left(x+\frac{x^3}{3}+\frac{x^5}{5}+\cdots\right) \quad -1<x<1. \quad (15)$$

2.6. Data Collection

An achievement test was employed to collect data on students' perceptions of the two methods. The test comprised five questions, each designed to assess the students' understanding and perception of the methods, following recommendations from previous studies [12][13]. The questions were as follows: (1) The method accurately describes the function ln; (2) The method is easy to remember; (3) The method facilitates manual calculations; (4) The method facilitates the creation of computer programs; and (5) The method aids in mastering the material. Each question was rated on a scale from 0 to 10, with 0 indicating strong disagreement and 10 indicating strong agreement. The purpose of the test was to evaluate students' perceptions of the methods' effectiveness, ease of use, and overall comprehensibility.

2.7. Data Analysis

The quantitative data obtained from the test were analyzed using a paired t-test to evaluate the mean difference. As the paired t-test is a parametric test, the assumption of normality was first examined using the Kolmogorov–Smirnov method. This test assesses whether the distribution of scores is consistent with a normal distribution. If the data were found not to be normally distributed, a non-parametric alternative, the Wilcoxon signed-rank test, was applied. The use of both tests was appropriate, as the data were derived from the same group of students who underwent the same intervention, making the methods suitable for paired data analysis. The paired t-test compares dependent samples and is formulated as follows:

$$t_c=\frac{\bar{d}}{SD/\sqrt{n}} \quad (16)$$

$$SD=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(d_i-\bar{d})^2} \quad (17)$$

where tc is the calculated t-value, $\bar{d}$ is the mean difference in perception scores between the Riemann integral and geometric series methods, SD is the standard deviation of the differences, n is the sample size, and d represents individual differences in perception scores. The p-value was calculated using the cumulative distribution function of the t-distribution, with statistical significance established at $p < 0.05$.

3. Results and Discussion

The students' perceptions of the Riemann integral and geometric series approaches in calculating the natural logarithm ln are presented in Table 1. The students perceived the Riemann integral as superior in describing the ln function, as well as being easier to remember and to use in developing computer programs. Each of these differences was statistically significant with $p<0.001$. Additionally, the Riemann integral was perceived as easier than the geometric series for manual calculation ($p=0.032$). However, the perceived level of difficulty for mastering the material was not significantly different between the two methods ($p=0.660$).

Table 1. Comparison of student perceptions of the Riemann integral and geometric series

Perception Test Results

Question	Mean±SD Riemann integral	Mean±SD Geometric series	tcount	p-value
The method clearly describes the ln function	8.30±1.14	5.96±1.33	5.655	<0.001
The formula is easy to remember	8.93±0.96	7.35±1.44	4.434	<0.001
The method is easy for manual calculations	7.01±1.42	7.93±1.45	2.295	0.032
The method is easy to implement in computer programs	7.70±0.89	5.87±1.07	8.332	<0.001
The method facilities mastery of the material	7.33±1.10	7.43±1.21	-0.447	0.660

Table 2 presents the process of calculating the tc value by comparing students' perceptions of the Riemann integral and geometric series methods. Using a significance level of $\alpha=0.05$ and 22 degrees of freedom, the critical t-value from the table is tt=2.07. Applying the paired sample t-test, the results indicate a statistically significant difference between the two methods (null hypothesis rejected because $tc \geq tt$). Therefore, the score derived using the Riemann integral development method is superior to that obtained through the geometric series method in describing the natural logarithm function (see Table 1).

Additionally, the Riemann integral formulation is reported to be easier to remember, more conducive to manual calculation by students, and more suitable for developing computer programs (see Table 2 for $tc \geq tt$, and Table 1 for comparative performance of the methods). In contrast, the paired sample t-test on the material mastery question showed no significant difference between the two methods (null hypothesis accepted because $-tt < tc < tt$), suggesting comparable student perceptions of material mastery for both approaches.

The five questions used in the study aimed to guide the acquisition of new knowledge and support the formation of generalizations. Inductive reasoning is widely acknowledged as a method and instrument for acquiring new knowledge [17]. However, the importance of the components of this type of mathematical thinking, particularly the observation of regularities, recognition of patterns, and formulation of generalizations, is often underestimated.

Implementation of Riemann Integrals in Natural Logarithmic Calculation

Riemann integrals are often preferred over geometric series for calculating natural logarithms (ln) due to their robustness and broad applicability in various mathematical contexts. The integration of Riemann's approach with the Fundamental Theorem of Calculus further enhances its utility, as it coherently links differentiation and integration, allowing for the efficient calculation of antiderivatives and definite integrals. In contrast, geometric sequences, though useful in specific scenarios, are limited in scope and application. A geometric series, which is the sum of terms in a geometric progression, is typically applied to sequences and series that converge under certain conditions. However, geometric series do not offer a general method for integration or for determining the area under a curve [25].

Conversely, Riemann integrals can accommodate a wider class of functions, making them a more powerful tool in mathematical analysis. Historically, the evolution of integration techniques, including the Riemann integral, has been motivated by the need to solve practical problems involving area, volume, and other geometric quantities [7]. This historical context emphasizes the fundamental importance of integration in mathematics, in contrast to the more narrowly applicable geometric sequences.

Riemann's systematic approach to integration, its alignment with foundational theorems, and its adaptability to various function types make it the preferred method for evaluating values such as ln over geometric series [30]. For instance, by using the Riemann integral, the properties of the natural logarithmic function can be demonstrated. Consider the following limit:

$$\lim_{n\to\infty}(1/(n+1)+1/(n+2)+1/(n+3)+\cdots+1/an)+(1/(n+1)+1/(n+2)+1/(n+3)+\cdots+1/abn) = \ln(ab)$$

$$=\ln(a)+\ln(b)$$

$$=\lim_{n\to\infty}(1/(n+1)+1/(n+2)+1/(n+3)+\cdots+1/an)+\lim_{n\to\infty}(1/(n+1)+1/(n+2)+1/(n+3)+\cdots+1/bn)$$

Such logarithmic properties, including $\ln(a)+\ln(b)=\ln(ab)$, cannot be derived using geometric series, underscoring the greater descriptive capability of Riemann integrals for the ln function.

Moreover, Riemann integrals allow explicit calculation of natural logarithmic values, which is not feasible through geometric series. For example, to compute ln(2) using a geometric series, one must determine the value of x that satisfies a specific condition (e.g., $x = 1/3$), and similarly for ln(3), $x = 1/2$. In contrast, the Riemann approach yields simpler and more intuitive formulas, as shown below:

$$\ln(2)=\lim_{n\to\infty}(1/(n+1)+1/(n+2)+1/(n+3)+\cdots+1/2n)$$

$$\ln(3)=\lim_{n\to\infty}(1/(n+1)+1/(n+2)+1/(n+3)+\cdots+1/3n)$$

$$\ln(4)=\lim_{n\to\infty}(1/(n+1)+1/(n+2)+1/(n+3)+\cdots+1/4n), \text{ etc.}$$

These series highlight the structured and memorable nature of logarithmic calculations using Riemann integrals. Although conceptually more complex, the Riemann integral is essential for understanding the framework of integration and addressing applied problems, despite students' frequent difficulties with its abstract nature [22].

In terms of manual computation, geometric series may seem simpler. For instance, when calculating ln(100), setting $x = 99/101$ and using $n=1000$ would only require summing 100 terms. In contrast, applying the Riemann method with $n=1000$ involves a summation of 100 x 1000-100=99,900 terms, significantly more than the geometric approach. However, for computational implementation, Riemann integration requires only a single function type and a looping structure, while geometric series demand preliminary determination of x (from Equation (15)), before the loop is executed.

While the geometric series approach offers a more straightforward formula for calculating ln values [2], the Riemann integral represents the foundational concept in integration, applicable not only to ln but to a broader class of functions. Observational findings indicate that students find both methods relatively accessible. With further exploration, students can develop a deeper conceptual understanding of mathematical relationships, thereby strengthening their cognitive connections across mathematical domains.

This comprehensive understanding forms a solid foundation for procedural mastery—an essential competence for addressing complex global challenges, such as those in sustainability, clean technology, and societal resilience. By equipping students with rigorous problem-solving capabilities and the ability to apply theoretical concepts in practical contexts, these mathematical methods contribute significantly to the development of innovative solutions aligned with global sustainability goals. This reflects the journal's commitment to advancing transformative research in pursuit of a sustainable future.

4. Conclusions

The findings of this study reveal that the Riemann integral method provides a robust and versatile framework for calculating the natural logarithm (ln) function. Students perceived this method as easier to describe, memorize, and apply in computational tasks, reflecting its utility in both theoretical and practical settings. While the geometric series method was preferred for manual calculations due to its simplicity, both methods were deemed equally effective in aiding students' mastery of the material. These insights underscore the importance of incorporating both approaches in mathematics education to equip students with diverse numerical skills adaptable to various contexts.

The broader implications of these findings extend beyond education into the realm of sustainable computational practices. Developing efficient algorithms based on the Riemann integral and geometric series methods can facilitate advancements in clean technologies and sustainable engineering. For instance, precise mathematical computations play a critical role in optimizing energy systems, improving material efficiency, and supporting innovations in resource management. By enhancing students' proficiency in such methods, mathematics education contributes to preparing the next generation of problem-solvers equipped to tackle sustainability challenges.

Future research should aim to refine these methods for improved accuracy and usability. A key area of exploration is the development of computational tools that leverage these methods to calculate ln values and other fundamental constants, such as π, with minimal error. Comparative studies assessing relative errors across different approaches can provide further insights into optimizing computational efficiency. Additionally, integrating these methods into software applications for modeling and simulations in engineering and environmental sciences would reinforce their practical relevance.

Author Statements:

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