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Research Article

Sensor Sensitivity based on exceptional points in Brillouin optomechanical system

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Abstract:

We propose a mass sensor based on exceptional points (EPs) engineered under synthetic magnetism in a Brillouin optomechanical system. This scheme consists of two optical modes and two mechanically coupled resonators. One mechanical resonator is subject to Backward Stimulated Brillouin Scattering (BSBS), which is critical for inducing an additional EP. We show that our system hosts two distinct EPs, the first is induced by the BSBS phenomenon while the second is common to optomechanical system under synthetic magnetism. Any perturbation to the system, such as a change in frequency, leads to a significant enhancement of sensitivity at both EPs. We find that the EP induced by the BSBS phenomenon provides an even greater sensitivity enhancement. Our proposal is a significant advancement over systems with a single EP, as it provides two distinct points of enhanced sensitivity, offering a broader operational range and improved performance. This work paves the way for more precise quantum measurements and quantum detection.

1. Introduction

Mass sensors are essential devices capable of detecting slight changes in mass. This capability is of paramount importance, with applications ranging from environmental monitoring [1], [2], [3] and biosensitivity [4], [5], [6] to precision medical diagnostics [7].The quest for enhanced sensitivity in these devices has led to significant innovation, particularly in the field of nanomechanical and optomechanical systems[8], [9], [10]. Traditional mass sensors, which typically operate on principles such as the detection of resonant frequency shift, often encounter fundamental constraints on their sensitivity[11], [12]. These limitations are mainly imposed by intrinsic factors such as thermal noise and various damping mechanisms. Overcoming these inherent limitations requires the development of new approaches capable of effectively amplifying the system response to small external perturbations. While traditional approaches [8], [13], [14] have pushed the boundaries of what is possible, they are fundamentally governed by the physics of Hermitian systems, where a linear relationship exists between the applied perturbation and the sensor response. This linear scale often presents a challenging ceiling in terms of achievable sensitivity, particularly in environments where ambient noise is a significant factor. A revolutionary paradigm shift has recently emerged with the exploration of non-Hermitian systems[15], [16], which has introduced a new and promising avenue for enhancing sensor performance. A key feature of these systems is the presence of exceptional points (Eps) [17], [18]. EPs are unique and unconventional degeneracies in the parameter space of a system, where eigenvalues coalesce and the corresponding eigenvectors merge. This simultaneous coalescence of eigenvalues and eigenvectors distinguishes EPs from the more common degeneracies encountered in Hermitian systems. At these highly specific EPs, the system response to an external perturbation is significantly amplified [19], [20], [21], [22]. Unlike the linear response observed in conventional Hermitian systems, the sensitivity in systems operating at an EP exhibits a square-root dependency for small perturbation [23] easily detectable, change in the system output. This phenomenon enables the creation of sensors that are orders of magnitude more sensitive than their classical counterparts.

Backward Stimulated Brillouin scattering (BSBS) [24], [25] is a powerful nonlinear optical effect where light from Optomechanical entanglement induced by backward stimulated Brillouin scatteringa laser interacts with the acoustic modes of a material, causing an increase in the mechanical dissipation rate of these modes. Although this increased dissipation may seem counterintuitive, it is highly beneficial for advanced quantum applications such as optomechanical entanglement [26], [27], [28] cooling [29], [30]and quantum squeezing [31], [32]. Beyond these applications, the strong light-matter coupling inherent to BSBS presents significant potential for enhancing sensor sensitivity. The nonlinear nature of the process can amplify the signal-to-noise ratio of a sensor, thereby enabling the creation of highly sensitive sensors [33], [34], [35]. The potential to combine BSBS with other advanced techniques, such as those based on EPs, is a new avenue for developing a new class of ultra-sensitive sensors.

In this paper, we combine the EP-sensor approach and the BSBS process in an optomechanical system to enhance the sensor sensitivity. Our benchmark system consists of two optical modes and two mechanically coupled resonators. The mechanical interaction is modulated by a phase θ By tunning this phase to $\theta = \frac{\pi}{2}(2n+1)$ being any integer, we enable the synthetic magnetism in our system, necessary to engineer the EP. The presence of the BSBS process induces a new EP in our system. Moreover, this EP offers a great sensitivity than the common EP. Our work provides an alternative way to enhance sensor sensitivity in optomechanical system, by taking advantage of joint effect of the synthetic magnetism and BSBS process.

The rest of the manuscript is organized as follow, section 2.1 presents the model and derives the dynamical equations. The enhancement of the sensor sensitivity is presented in section 2. Our work is concluded in section 3.

2. Material and Methods

Our benchmark optomechanical system consists of two optical modes and two mechanical resonators. The system unique properties stem from few key features, one of the two mechanical resonators is subject to BSBS process, which enhances its mechanical decay rate. The two resonators are coupled through a phonon-hopping interaction, with a coupling rate (J_m) that can be modulated by a phase (θ). This setup is represented in **Figure 1**

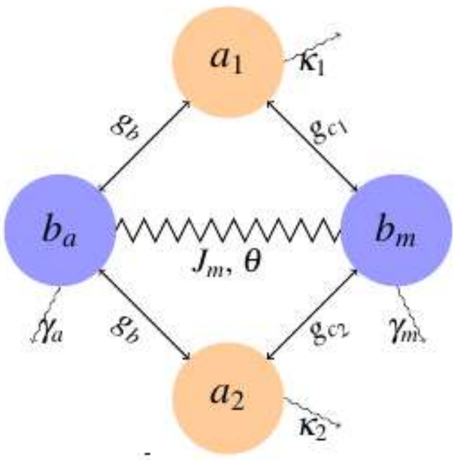


Figure 1. *Sketch of our benchmark system*

In the frame rotating at the driving frequencies, our system is described by the following Hamiltonian $\hbar = 1$,

$$H = H_0 + H_{OM} + H_{BSBS} + H_{int} + H_{Drive} \quad (1)$$

Where

$$H_0 = \sum_{j=1,2} -\Delta_j \, a^\dagger{}_j a_j + \Delta_a b_a^\dagger b_a + \omega_m b_m^\dagger b_m \quad (2)$$

$$H_{OM} = -\sum_{j=1,2} g_{cj} a_j^\dagger a_j (b_m + b_m^\dagger) \quad (3)$$

$$H_{BSBS} = -g_b (a_1^\dagger a_2 b_a + a_1 a_2^\dagger b_a^\dagger) \quad (4)$$

$$H_{int} = J_m (e^{i\theta} b_a^\dagger b_m + e^{-i\theta} b_a^\dagger b_m) \quad (5)$$

$$H_{drive} = \sum_{j=1,2} i E_j (a_j^\dagger e^{-i\omega_{pj} t} - a_j e^{i\omega_{pj} t}) \quad (6)$$

In the above Hamiltonian, $H_0, H_{OM}, H_{BSBS}, H_{int}$ $and\ H_{drive}$ represent respectively the free Hamiltonian, the optomechanical interaction, the BSBS process, the phonon-phonon interaction and the driving fields. The terms $a_j(a_j^\dagger), b_a(b_a^\dagger)$ $and\ b_m(b_m^\dagger)$ are the annihilation (creation) operators of the j^{th} optical mode (with frequency ω_{c_j}), the acoustic mode (with frequency ω_a) and the mechanical mode (with frequency ω_mThe optomechanical couplings between the j^{th} driven field and one of the mechanical resonators, are captured by g_{c_j}. The j^{th} driving field is E_j and the frequency detuning are defined by $\Delta_j = \omega_{p_j} - \omega_{c_j}$ and $\Delta_j = \omega_a + \omega_{p_2} - \omega_{p_1}$.

The dynamical equations of our system are derived from the Heisenberg equation, which yield,

$$\begin{cases} \dot{\alpha}_1 = \left(i\Delta_1 + ig_{c_1}(b_m + b^{\dagger}{}_m) - \frac{\kappa_1}{2}\right)a_1 + ig_b a_2 b_a \\ \qquad +E_1 + \sqrt{\kappa_1}a_1^{in} \\ \dot{\alpha}_2 = \left(i\Delta_2 + ig_{c_2}(b_m + b^{\dagger}{}_m) - \frac{\kappa_2}{2}\right)a_2 + ig_b a_1 b_a^{\dagger} \\ \qquad +E_2 + \sqrt{\kappa_2}a_2^{in} \\ \dot{b}_a = -\left(\frac{\gamma_a}{2} + i\Delta_a\right)b_a - iJ_m e^{i\theta}b_m + ig_b a_1 a_2^{\dagger} \\ \qquad +\sqrt{\gamma_a}b_a^{in} \\ \dot{b}_m = -\left(\frac{\gamma_m}{2} + i\omega_m\right)b_m + ig_{c_1}a_1^{\dagger}a_1 + ig_{c_2}a_2^{\dagger}a_2 \\ \qquad -iJ_m e^{-i\theta}b_a + \sqrt{\gamma_m}b_m^{in} \end{cases} \quad (7)$$

where κ_j, γ_a and γ_m are the dissipations related to each mode of our system. The quantities a_j^{in}, b_a^{in} and b_m^{in} stand for the thermal quantum noise operators. These operators are characterized by zero mean value, and the following correlation functions,

$$\langle a_j^{in}(t)a_j^{in\dagger}(t')\rangle = \delta(t-t'), \langle a_j^{in\dagger}(t)a_j^{in}(t')\rangle = 0 \quad (8)$$
$$\langle b_a^{in}(t)b_a^{in\dagger}(t')\rangle = \delta(t-t'), \langle b_a^{in\dagger}(t)b_a^{in}(t')\rangle = 0 \quad (9)$$
$$\langle b_m^{in}(t)b_m^{in\dagger}(t')\rangle = (n_m+1)\delta(t-t'), \quad (10)$$
$$\langle b_m^{in\dagger}(t)b_m^{in}(t')\rangle = n_m\delta(t-t'), \quad (11)$$

where n_m is the thermal phonon occupation number of the mechanical resonator b_m that is defined as $n_m = \left[\exp\left(\frac{\hbar\omega_m}{k_{bT}}\right) - 1\right]^{-1}$, with k_b the Boltzmann constant and T the bath temperature.

Under the assumption that the driving field a_2 is strong enough to allow us to neglect its optomechanical coupling to resonator b_m, we can simplify the full system description, leading to the following equations,

$$\begin{cases} \dot{\alpha}_1 = \left(i\Delta_1 + ig_{c_1}(b_m + b^{\dagger}{}_m) - \frac{\kappa_1}{2}\right)a_1 + iG_b b_a + E_1 \\ \qquad +\sqrt{\kappa_1}a_1^{in} \\ \dot{b}_a = -\left(\frac{\gamma_a}{2} + i\Delta_a\right)b_a - iJ_m e^{i\theta}b_m + iG_b a_1 + +\sqrt{\gamma_a}b_a^{in} \\ \dot{b}_m = -\left(\frac{\gamma_m}{2} + i\omega_m\right)b_m + ig_{c_1}a_1^{\dagger}a_1 - iJ_m e^{-i\theta}b_a \\ \qquad +\sqrt{\gamma_m}b_m^{in} \end{cases} \quad (12)$$

with the Brillouin coupling strength

$G_b = g_b\alpha_2 = \frac{-g_b E_2}{i\Delta_2 - \frac{\kappa_2}{2}}$.

To accurately measure the sensitivity of our system, we must compute the eigenvalues of the effective system, which dictate its response to perturbations. The process begins with the QLEs (Eq. 12), which are nonlinear and describe the evolution of the quantum operators. To simplify these equations, we use a standard linearization procedure by splitting each operator into its classical mean value and a small quantum fluctuation. By doing this, we obtain a separate set of equations for the mean values,

$$\begin{cases} \dot{\alpha}_1 = \left(i\tilde{\Delta} - \frac{\kappa_1}{2}\right)\alpha_1 + iG_b\beta_a + \sqrt{\kappa_1}\alpha_1^{in} \\ \dot{\beta}_a = -\left(\frac{\gamma_a}{2} + i\Delta_a\right)\beta_a - iJ_m e^{i\theta}\beta_m + iG_b\alpha_1 \\ \dot{\beta}_m = -\left(\frac{\gamma_m}{2} + i\omega_m\right)\beta_m - iJ_m e^{-i\theta}\beta_a + ig_{c_1}|\alpha_1|^2 \end{cases} \quad (13)$$

and a set of the fluctuation dynamical equations,

$$\begin{cases} \delta\dot{\alpha}_1 = \left(i\tilde{\Delta} - \frac{\kappa_1}{2}\right)\delta a_1 + ig_{c_1}(\delta b_m + \delta b_m^{\dagger})\alpha_1 + iG_b\delta b_a \\ \qquad +\sqrt{\kappa_1}a_1^{in} \\ \delta\dot{b}_a = -\left(i\Delta_a + \frac{\gamma_a}{2}\right)\delta b_a - iJ_m e^{i\theta}\delta b_m + iG_b\delta a_1 + \sqrt{\gamma_a}b_a^{in}, \\ \delta\dot{b}_m = -\left(i\omega_m + \frac{\gamma_m}{2}\right)\delta b_m - iJ_m e^{-i\theta}\delta b_a + ig_{c_1}(\alpha_1^*\delta a_1 \\ \qquad +\alpha_1\delta a_1^{\dagger}) + \sqrt{\gamma_m}b_m^{in} \end{cases} \quad (14)$$

where $\tilde{\Delta} = \Delta_1 + g_{c_1}(\beta_m + \beta_m^{\dagger})$ is the effective detuning.

By using adiabatic elimination, we simplify the dynamics of the system by eliminating the rapidly decaying optical field, which is much faster than the mechanical modes. This results in a simpler system, described solely by the mechanical resonators, thereby facilitating the analysis of key parameters such as sensitivity,

$$\begin{cases} \delta\dot{b}_a = -\left(i\Delta_a + \frac{\gamma_a + \Gamma_a}{2}\right)\delta b_a - \left(iJ_m e^{i\theta} + \frac{\Gamma_{am}}{2}\right)\delta b_m \\ \qquad +i\sqrt{\Gamma_a}a_1^{in} + \sqrt{\gamma_a}b_a^{in} \\ \delta\dot{b}_m = -\left(iJ_m e^{-i\theta} + \frac{\Gamma_{am}^*}{2}\right)\delta b_a - \left(i\omega_m + \frac{\gamma_m + \Gamma_m}{2}\right)\delta b_m \\ \qquad +i\sqrt{\Gamma_m}a_1^{in} + \sqrt{\gamma_m}b_m^{in} \end{cases}, \quad (15)$$

where $\Gamma_a = \frac{4|G_b|^2}{\kappa_1}$, $\Gamma_m = \frac{4|G_c|^2}{\kappa_1}$ and $\Gamma_{am} = \frac{4G_bG_c}{\kappa_1}$.

We can rewrite this system by considering $u = (\delta b_a, \delta b_m)^{\top}$ and the effective Hamiltonian H_{eff},

$$\dot{u} = -iH_{eff}u. \quad (16)$$

H_{eff} is defined as,

$$H_{eff} = \begin{pmatrix} \Delta_a - i\frac{\gamma_a^{eff}}{2} & J_m e^{i\theta} - i\frac{\Gamma_{am}}{2} \\ J_m e^{-i\theta} - i\frac{\Gamma_{am}^*}{2} & \omega_m - i\frac{\gamma_m^{eff}}{2} \end{pmatrix}, \quad (17)$$

where $\gamma_a^{eff} = \gamma_a + \Gamma_a$ and $\gamma_m^{eff} = \gamma_m + \Gamma_m$.

The eigenvalues of the above Hamiltonian is given by

$$\lambda_\pm \quad = \frac{1}{2}(\Delta_a + \omega_m) - \frac{i}{4}\left(\gamma_a^{eff} + \gamma_m^{eff}\right) \pm \frac{\sigma}{4} \qquad (18)$$

Where

$$\sigma = \sqrt{\Upsilon - 16 i J_m \Gamma_{am} \cos(\theta)}, \qquad (19)$$

with $\Upsilon = \left(2\Delta\omega + i\Delta\gamma^{eff}\right)^2 + 16J_m^2 - 4|\Gamma_{am}|^2$, $\Delta\omega = \Delta_a - \omega_m$ and $\Delta\gamma^{eff} = \gamma_m^{eff} - \gamma_a^{eff}$.

We consider the coupling strengths G_c and G_b as real.

The real and imaginary parts of these eigenvalues represent, respectively, the eigenfrequencies ($\omega_\pm$) and eigendampings ($\gamma_\pm$) of the system. At an EP, a unique topological feature of non-Hermitian systems, these eigenfrequencies and eigendampings undergo a simultaneous coalescence.

2.1. Sensor sensitivity of the system

Before investigating our sensor sensitivity, we must first locate its EPs ($\sigma = 0$). The analysis of the eigenvalues in **Hata! Başvuru kaynağı bulunamadı.**, shows that a standard optomechanical system under synthetic magnetism has a single EP (black) corresponding to $J_m = \frac{\gamma_m^{eff} - \gamma_a}{4}$. However, when we introduce the BSBS process, the system gives rise to two distinct EPs (blue) corresponding to the two solution of the characteristic equation in Appendix A. A new BSBS-induced EP at a lower optomechanical coupling strength ($G_c = 0.01049\omega_m$), while the second ($G_c = 0.06110\omega_m$) is a more conventional EP. This dual-EP configuration is especially valuable because it gives us two opportunities to explore and optimize sensor performance, providing a more comprehensive platform for analysis than a single-EP system. The presence of the BSBS process in our system offers a significant advantage for engineering an EP. In contrast to conventional methods that require a large optomechanical coupling, the BSBS process allows the system to reach an EP with a substantially smaller coupling strength. The phase θ of the mechanical coupling is a critical parameter for our system, playing a fundamental role in the formation and location of EPs, as illustrated in Figure . Our analysis reveals a strong dependency of the EPs on this phase, with the eigenfrequencies and eigendampings coalescing precisely when $\theta = \frac{\pi}{2}(2n+1)$, where n is an integer. This specific

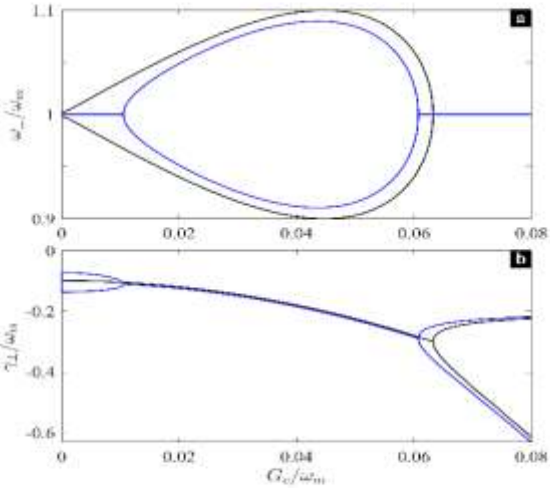


Figure 2.*(a) Eigenfrequencies and (b) eigendampings of the system without BSBS process (black) and with BSBS process (blue). Parameters used are* $\Delta_a = \omega_m, \kappa_j = 2 \times 10^{-2}\,\omega_m, \gamma_a = 0.4\,\omega_m, \gamma_m = 10^{-4}\omega_m, \kappa_j = 2 \times 10^{-2}\omega_m, g_{c_j} 10^{-4}\ \ \omega_m, J_m = 10\omega_m$

phase modulation acts as a key for manipulating the non-Hermitian dynamics of the system, directly influencing the conditions under which EP induced sensitivity enhancement can be achieved. This finding underscores the importance of precise phase control in designing and operating high-performance sensors based on these principles.

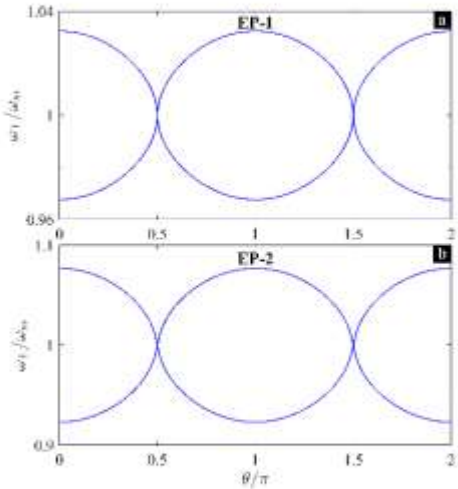


Figure 3. *Eigenfrequencies versus the phase* θ *for (a) EP-1 at* $G_c = 0.01049\omega_m$ *and (b) EP-2 at* $G_c = 0.06110\omega_m$*, in presence of the BSBS process (*$G_b = 10^{-2}\ \omega_m$*). The used parameters are the same as those in* ***Figure 2***.

W

is introduced to our system, it causes a small shift in the mechanical frequency, represented by ϵ. The eigenvalues of the system after the perturbation is written as,

$$\lambda_\pm^p \quad = \frac{1}{2}(\Delta_a + \omega_m + \epsilon) - \frac{i}{4}\left(\gamma_a^{eff} + \gamma_m^{eff}\right) \pm \frac{\sigma_p}{4}, \qquad (20)$$

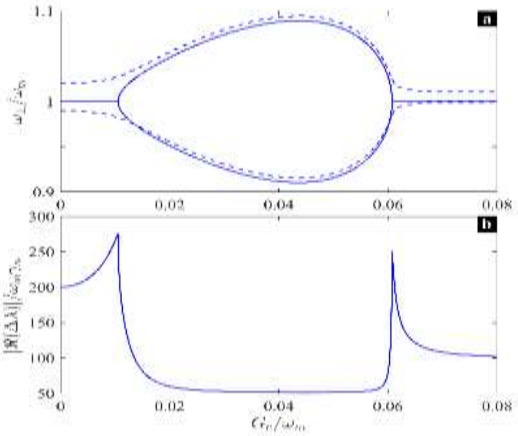
Where

$$\sigma_p = \sqrt{\Upsilon_p - 16iJ_m\Gamma_{am}\cos(\theta)}, \qquad (21)$$

with $\Upsilon_p = (2\Delta\omega + i\Delta\gamma^{eff} - 2\epsilon)^2 + 16J_m^2 - 4|\Gamma_{am}|^2$.

The introduction of the perturbation fundamentally alters the Hamiltonian, which in turn acts to lift the degeneracy at the EPs as shown in **Hata! Başvuru kaynağı bulunamadı.**a. This action causes the eigenfrequencies and eigendampings to undergo a measurable splitting. Unlike the response of conventional sensors, this splitting allows for a significantly amplified readout. To quantitatively evaluate the sensor efficiency, we formally define its sensitivity as the magnitude of this eigenvalue splitting. Specifically, we measure the absolute difference between the perturbed and unperturbed eigenvalues,

$$|\Re(\Delta\lambda)| = |\Re(\lambda_\pm^p - \lambda_\pm)| \qquad (22)$$



*l l sensitivity after a perturbation (dashed curve) in presence of the BSBS process. The perturbation strength is $\varepsilon = 10^{-2}\omega_m$ and the other parameters are the same as those in **Figure 2**.*

This quantity provides a clear and direct measure of the sensor performance, as a larger eigenvalue splitting for a given perturbation indicates a higher level of sensitivity. By computing this quantity, we observe two peaks of sensitivity corresponding to the two EPs as depicted in **Hata! Başvuru kaynağı bulunamadı.**b. Moreover, we notice that the EP induced by the BSBS process is more sensible than the standard EP. This remark highlights the important role of the BSBS process. Rather than having only one point of sensitivity measure like in the standard EP sensor, we have the possibility to choose one of the two points for sensing.A similar analysis is conducted to examine the behavior of the system sensitivity with respect to the phase θ. As depicted in **Hata! Başvuru kaynağı bulunamadı.**, the perturbation causes the splitting of the eigenfrequencies and the sensor sensitivity peaks precisely at phase values of $\theta = \frac{\pi}{2}(2n+1)$, a condition that is a signature of synthetic magnetism. This indicates that the system performance is highly dependent on precise phase control. However, the sensor sensitivity at the BSBS-induced EP is consistently greater than that of the standard EP. This crucial distinction implies that the BSBS mechanism provides a more robust sensing platform.

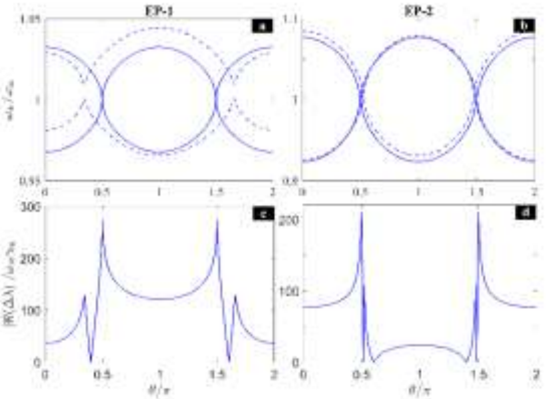


Figure 5. *(a) Eigenfrequencies splitting and (b) the resulted sensitivity after a perturbation (dashed curve) at EPs, in presence of the BSBS process. The other parameters are the same as those in Figure 2.*

Fig. 6 an illustrates the relationship between the sensor sensitivity and the strength of the perturbation at both exceptional points (EPs). The analysis reveals that the sensitivities at both EPs follow a similar evolution as the perturbation strength increases, the overall sensitivity of the sensor also increases. A closer examination of the regime of small perturbations, yields a significant and compelling result. The sensitivity at the first EP is approximately double the sensitivity observed at the second EP. This remarkable enhancement is directly attributed to the BSBS process. This observation is further corroborated when investigating the enhancement factor defined as,

$$\eta = \frac{|\Re(\Delta\lambda)|}{\epsilon}. \qquad (23)$$

For small perturbations, this enhancement factor is significantly greater at the BSBS induced EP than at the standard EP. The difference between the two is substantial but decreases with an increase in perturbation strength, tending towards a sensitivity equal to that of conventional sensors for significant perturbations.

3. Conclusions

This work investigated the sensor sensitivity at EPs in a Brillouin optomechanical system. Our benchmark system is made of two mechanically coupled resonators and two optical modes. The mechanical coupling, which is captured by the

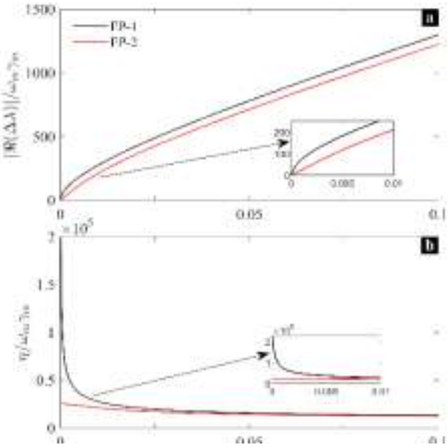


Figure 6. (a) Sensor sensitivity and (b) enhancement factor versus the perturbation strength. The used parameters are the same as in Figure 2.

phonon hopping rate J_m, is modulated through a phase θ that is used to engineer an EP in our system. By introducing the BSBS interation between the two optical modes and on of the resonator, we induce an additional EP. The sensor sensitivity is greatly enhanced at the EPs corresponding to the phase $\theta = \frac{\pi}{2(2n+1)}$. Moreover, the BSBS induced EP exhibits a greater sensor sensitivity than the standard EP. This approach represents a more flexible scheme to engineer EPs and enhance the sensor sensitivity. This work paves the way for the design of more flexible and efficient optomechanical sensors.

Appendix A: Characteristic equation for Eps in presence of BSBS process

In the presence of the BSBS process, the condition for engineering EPs is

$$\sigma = 0, \quad (A1)$$

$$\Upsilon - 16iJ_m\Gamma_{am}\cos(\theta) = 0, \quad (A2)$$

$$(2\Delta\omega + i\Delta\gamma^{eff})^2 + 16J_m^2 - 4|\Gamma_{am}|^2 - 16iJ_m\Gamma_{am}\cos(\theta) = 0. \quad (A3)$$

By taking into consideration the parameters $\Delta_a = \omega_m$ and $\theta = \frac{\pi}{2}$, we simplify previous equation to,

$$-(\Delta\gamma^{eff})^2 + 16J_m^2 - 4|\Gamma_{am}|^2 = 0 \quad (A4)$$

By substituting the expression of the effective parameters, we obtain the final characteristic equation,

$$X^2 + AX + B, \quad (A5)$$

with $X = |G_c|^2, A = 2|G_b|^2 + \frac{\kappa_1\Delta\gamma^2}{2}, B = \frac{\kappa_1{}^2\Delta\gamma^2}{16} + |G_b|^4 - \frac{\kappa_1\Delta\gamma^2|G_b|^2}{2} - \kappa_1^2 J_m^2$ and $\Delta\gamma = \gamma_m - \gamma_a$.

Author Statements:

- **Ethical approval:** The conducted research is not related to either human or animal use.
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