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Research Article

Numerical Analysis of Slope Stability under Surcharge Effects Using Convex Crossover Optimization

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Abstract:

An important problem in geotechnical engineering is slope stability analysis, yet it remains highly complex due to the nonlinear nature of failure mechanisms. The evaluation requires determining the critical failure surface (CFS) and the minimum factor of safety (FOS), which poses a challenging optimization problem. In this study, the limit equilibrium method (LEM), based on the Fellenius approach, is applied to assess slope stability under varying surcharge conditions, including surcharge intensity, position, and width. Results show that higher surcharge intensity combined with reduced position distance and width significantly undermines slope stability. Traditional optimization algorithms are often inadequate for such problems, as they rely on initial guesses and may converge only to local optima. To address this, a Multi-Parametric Genetic Algorithm (MPGA) with a Convex Crossover (CC) operator is developed. The CC operator parameters are calibrated using benchmark test functions, while a time-dependent decay factor is incorporated to improve variable interaction and prevent premature convergence. The proposed MPGA consistently identifies more accurate critical failure surfaces and provides lower safety factors compared to conventional approaches. Overall, the method demonstrates superior efficiency and robustness, offering a reliable tool for slope stability optimization under surcharge conditions.

1. Introduction

One of the most important problems in geotechnical engineering is analyzing slope stability, as it directly affects the safety and performance of earth structures such as dams, embankments, and cut slopes. This problem has attracted significant attention from researchers because slope instability can occur in projects involving excavation, loading, or undercutting of natural slopes. The key objective in slope stability assessment is the determination of the critical slip surface (CFS), i.e., the slip surface associated with the minimum factor of safety (FOS). This problem is often formulated using principles derived from the calculus of variations [1–2].

Unstable slope conditions may arise due to various factors such as excessive surcharge loading (P), intense rainfall, or dynamic forces (e.g., earthquakes). The most widely used techniques for slope stability assessment are the classical limit equilibrium methods (LEM) [3–8]. These methods are based on dividing the potential sliding mass into a finite number of vertical slices and applying equilibrium equations of forces and moments to estimate the FOS. The next step involves identifying a critical slip surface, either with a predefined or general shape that minimizes the FOS. However, as noted by Fredlund [9], several challenges arise when applying LEM, including assumptions about inter-slice forces, the use of predefined slip surfaces, and possible inaccuracies in estimating FOS.

To overcome these limitations, numerical approaches such as the *finite element method (FEM)* using the strength reduction technique have been developed [10–14]. FEM provides additional complications, such as the need to define the stress–strain behavior of soils, but it does not require assumptions about the geometry of the slip surface like LEM does. The evaluation of slope stability has also made extensive use of the finite difference technique (FDM) [15]. Baker et al. [16] further applied the calculus of variations to the equilibrium problem and showed that the minimum FOS must occur along slip surfaces possessing specific geometric properties. Traditional trial-and-error techniques are often inadequate because of the complexity of the solution space and the presence of multiple local minima [17]. The FOS is generally a multimodal and non-smooth function, influenced by variations in soil parameters, loading conditions, and external forces. Under such conditions, classical methods may converge to local minima rather than the global minimum, leading to non-optimal solutions. To address this, stochastic and metaheuristic optimization methods have been increasingly employed in slope stability analysis, as they provide robust mechanisms for exploring complex search spaces.

Numerous metaheuristic algorithms have been used in recent research to address this issue. For instance, Goh [19] used a *Genetic Algorithm (GA)* to locate the CFS, demonstrating that GA can accurately identify slip surfaces consistent with actual failure conditions. Similarly, Zolfaghari et al. [20] combined GA with the Morgenstern–Price method, showing that while the choice of slip surface shape (circular vs. non-circular) has little effect on homogeneous slopes, it significantly affects layered slopes. Chen [21] applied a simulated annealing approach, while Li et al. [22] introduced a real-coded GA validated with case studies. Shinoda et al. [23] used Particle Swarm Optimization (PSO), whereas Khajehzadeh et al. [24–25] proposed a novel Gravitational Search Algorithm (GSA). Other techniques include Tabu Search (TS) [26], Fireworks Algorithm (FWA) [32], Black Hole Algorithm (BHA) [33], Immense Evolutionary Programming (IEP) [34]. Differential Evolution (DE) [35], Biogeography-Based Optimization (BBO) [36], Cuckoo Search (CS) [37], Improved Radial Movement Optimization (IRMO) [38], Genetic Programming (GP) [39], Ant Colony Optimization (ACO) [40], Imperialistic Competitive Algorithm (ICA) [42], as well as hybrid approaches combining neural networks with metaheuristics [44–47]. Fuzzy logic [48] and probabilistic approaches such as the *reliability* index method [49] have also been applied. Cheng et al. [50] compared six metaheuristic methods and concluded that no single algorithm consistently outperforms the others, as each has its strengths and limitations.

Despite their effectiveness, metaheuristic algorithms also face challenges, such as parameter tuning and the need for large numbers of iterations. Nonetheless, they present two key advantages: (i) efficient information-sharing mechanisms that often accelerate convergence, and (ii) reduced likelihood of entrapment in local optima.

In this study, we propose the use of the Multi-Parametric Genetic Algorithm (MPGA), based on Convex Crossover (CC), to identify the critical circular slip surface (CFS).(MPGA) is especially appealing due to its straightforward implementation and potent capacity to resolve intricate nonlinear optimization issues. The Fellenius method—a classical limit equilibrium approach is adopted, where the FOS is expressed as an objective function to be minimized. Using benchmark problems from the literature, the suggested algorithm's performance is assessed. A Neural Net Fitting model is used to further validate the outcomes. The outcomes demonstrate that the proposed MPGA provides superior accuracy and robustness compared to several conventional approaches for locating the CFS in homogeneous soil slopes. To the best of our knowledge, the current issue has not been tackled before in literature.

This paper's remaining sections are organized as follows:

• The methodological foundation for determining CFS is presented in **Section 2.**

- **Section 3** introduces the MPGA algorithm and its operators.
- **Section 4** reports benchmark case studies used to validate the approach.
- **Section 5** provides concluding remarks and discusses the main findings.

2. Methodology

2.1Slope stability analysis

Several conventional approaches can be employed for slope stability assessment, such as numerical, kinematic, probabilistic, limit equilibrium, and empirical methods [52, 9]. Among these, the limit equilibrium method (LEM) remains the most commonly adopted analytical tool in geotechnical engineering [24–25, 42, 9, 54], where the factor of safety is usually determined according to Mohr–Coulomb failure criterion and the method of slices, which involves identifying the critical failure surface (CFS) through a series of iterations, are used in the current work. For circular slip surfaces, the center coordinates C(x_0, y_0) and the radius R are systematically varied until the configuration corresponding to the minimum factor of safety (FOS) is obtained. Because the soil mass above the sliding surface is divided into vertical slices, the equilibrium can be stated as the balance between the driving forces acting on each slice and the resisting shear forces. In the proposed procedure, only plausible slip surfaces are considered, while unrealistic configurations that yield invalid or unreasonable FOS values are excluded from the analysis. The general procedure for evolving and locating the failure surface is presented in Fig. 1.

2.2Generating circular slip surface

This paper focuses solely on two-dimensional slope stability problems, assuming a Cartesian coordinate system as the frame of reference: **OXY.** The geometry of a slope's slip surface affects its safety factor (FOS).A acceptable slip surface usually stretches from the slope's top to its toe, is concave, and has a coordinate that increases or decreases monotonically. One classification for the slip surface is circular, and the former is a particular instance of the latter. In this part, we introduce a novel approach to creating slip surfaces on slopes. Finding the critical failure surface (CFS), or the surface along which a soil mass is most likely to fail, is the primary goal. The factor of safety (FOS), which is connected to a certain possible slip surface, measures this chance of failure. The polyline that represents this slip surface is made up of N points$[P0, P1, \dots, Pn-1]$. every kinematic and geometric limitation on this function must be met [55]. The slip surface is handled as a function here:

$$FOS = F\ (P).$$

Slip surfaces are commonly classified as circular or non-circular, with the circular geometry representing a special case of the latter. The methodology used here is intended to find the critical failure surface (CFS), which is the surface along which slope failure is most likely to occur, and to create probable slip surfaces. The likelihood of failure is expressed by the factor of safety (FOS), evaluated for each candidate surface. The slip surface is modeled as a polyline consisting of N points [P0, P1… Pn-1], subject to both kinematic and geometric constraints [55]. In this formulation, the slip surface is expressed as a function of the solution vector P, such that FOS = F(P). The positional vector matrix (P) represents the polyline describing the candidate slip surface, while the function F(P), as defined in Equation (1), maps the polyline coordinates to a scalar FOS value, thereby quantifying slope stability [50, 56].

$$P = [P0; P1; \dots; Pn-1] = \begin{bmatrix} P_0 = slope(t_0) = (x_0\,, y_0) \\ P_1 = (x_1\,, y_1) \\ P_2 = (x_2\,, y_2) \\ P_i = (x_i\,, y_i) \\ P_{i-2} = (x_{i-2}\,, y_{i-2}) \\ P_{i-1} = slope(t_{i-1}) = (x_{i-1}\,, y_{i-1}) \end{bmatrix} \quad .1$$

The function **F(P)** is generally discontinuous and multimodal, even when all geometric and kinematic constraints are satisfied. As shown by Chen et al. [51], multiple local minima can exist within the solution space. Thus, **F(P)** is inherently non-smooth, corresponding to an NP-complete problem, which justifies treating slope stability analysis as a global optimization problem [50, 51].

2.3Constraints and Bounds

Initially, the slope surface is parameterized throughout the interval from right to left [0, 1], and as such : n a Cartesian coordinate system (X–Y plane) illustrated in Fig. 2, the slip surface can be expressed as y=S(x), whereas the bedrock profile is given by y=R(x). To ensure that a potential failure surface satisfies both geometric and kinematic conditions, specific constraints must be imposed so that the system remains in equilibrium. These constraints determine the feasible domains of the 2n−2 control variables [57]. For the parameterization, the slope surface is defined from right to left within the interval [0,1]. Accordingly:

$$\{t_0\,, t_{i-1} |\, t_0 < t_{i-1} \in [0,1]\ \} \quad .2$$

$$P_0 = slope(t_0) = (x_0\,, y_0) \quad .3$$

$$P_{n-1} = slope(t_{n-1}) = (x_{n-1}\,, y_{n-1}) \quad .4$$

2.3.1 Bounds for x-coordinates

n−2 slices are produced by uniformly partitioning the failure surface, which is represented by n vertices, along the interval from x0 to xn-1. This slice establishes the matching set of x-coordinates so that:

$$x_i > x_j \mid j > i, \forall x_i = 1,2,3,\dots,n-2 \; \& x_j = 1,2,3,\dots,n-1 \quad .5$$

Where:

$$x_i \in (x_{n-1}, x_{n-1} - \frac{x_{n-1} - x_0}{n-2}) \quad .6$$

The spacing between each pair of consecutive vertices (P0, P1… Pn-1) is identical, such that:

$$(x_i - x_{n+1}) = (x_{i+1} - x_{n+2}) = \frac{x_{n-1} - x_0}{n-2} \quad .7$$

Where: $i \in [0, i-1]$

2.1.1 Bounds for y-coordinates

The slope height and slope base, which are readily retrieved from the slope geometry, immediately determine the control variables y0 and yn-1. On the other hand, to precisely specify the geometry of the slip surface, the remaining 2n-2 variables need to be optimized:

$$\{x_0, (x_1, y_1), \dots, (x_i, y_i), \dots, (x_{n-2}, y_{n-2}), x_{n-1}\} \quad .8$$

Will be derived according to the coordinate system used to define the slip surface. Cheng [50] proposed that if the values of a1 to xn-1 are defined, then the corresponding lower and upper bounds for (y2,…,yn) can based on the slope geometry and bedrock profile [58,59]. The bounds on the control variables should be dynamic to ensure an admissible surface. The bounds for the y0 coordinate given by:

$$y_0 \in \{(R_x, S_x)\} \quad .9$$

The vertical line x=x_2 is crossed at point H by the line connecting points P_{n-1} and P_1, with a matching y coordinate, y_H. Likewise, when the line passing through P_0 and P_1 is extended, it intersects the line x=x_2 at point G, where y coordinate y_G is used to calculate. Thus, (y_{2min}, y_{2max}) ane be calculated as follows:

$$y_{2min} = \max(y_G, R(x_3) \; \& y_{2max} = \min(y_H, y_0(x_3)) \quad .10$$

Similarly, the bound on the y coordinates of each point on the slip surface, i.e., are defined as follows$\{y_{1min}, y_{1max}, \cdots, y_{n-1min}, y_{n-1max}\}$. Shown in fig.2 , can be determined by the following relation:

$$y_i \in ((\max(R_{xi}), Y_{imin}), (\min(S_{xi}), Y_{imax})) \quad .11$$

$$Y_{imin} = y_{i-1} + (\frac{y_{i-1} - y_{i-2}}{x_{i-1} - x_{i-2}})(x_i - x_{i-1}) \quad .12$$

$$Y_{imax} = y_{i-1} + (\frac{y_{n-1} - y_{i-1}}{x_{n-1} - x_{i-2}})(x_i - x_{i-1}) \quad .13$$

2.4 Modeling of the objective function

In this work, an objective function that has been specially modified for use with the MPGA algorithm is presented. The factor of safety (FOS) is computed using the Fellenius method [3, 27,53], where the corresponding expression serves as the fitness function of the optimization procedure. In this approach, slope failure is assumed to occur along a circular slip surface. The interstice forces E_1, E_2 and X_1, X_2, illustrated in Fig. 3, are neglected—a simplification that remains valid under varying surcharge conditions, including changes in load magnitude, position (distance from the crest), and width. To characterize a circular slip surface, the circle's center C(x_0,y_0) and radius R must be specified, which are determined at the intersection points between the slope boundary and the circular arc. The potential slip surface is represented by arc AB, defined by the center O and radius R. The entire slope mass (ABCD) is then discretized into n slices, as shown in Fig. 3, enabling the computation of both the total resisting force (F_r) and the total driving force (F_d). Based on these values, the factor of safety (FOS) is finally expressed in the Fellenius method as:

$$\boldsymbol{FOS} = \sum_{i=1}^{n} \frac{\boldsymbol{Fr}}{\boldsymbol{Fd}} = \frac{\sum_{i=1}^{n}[\boldsymbol{c'_i l_i} + (\boldsymbol{W_i cos(\alpha)} - \boldsymbol{U_i l_i})\boldsymbol{tan(\emptyset'_i)}]}{\sum_{i=1}^{n} \boldsymbol{W_i sin(\alpha_i)}} \quad .14$$

$$U = r_u \gamma_w h_w \Rightarrow r_u = \frac{U}{\gamma_w h_w} \quad .15$$

In the final step, the (MPGA) algorithm is employed to minimize the objective function. During each iteration, the circle parameters—namely the center coordinates C(x_0, y_0) and the radius R are updated until the slip surface that yields the lowest factor of safety (FOS) is detected.

$$\boldsymbol{FOS} = \sum_{i=1}^{n} \frac{\boldsymbol{Fr}}{\boldsymbol{Fd}} = \frac{\sum_{i=1}^{n}[\boldsymbol{c'_i l_i} + (\boldsymbol{W_i} + \boldsymbol{P} * \boldsymbol{b})\boldsymbol{cos(\alpha)} - \boldsymbol{U_i l_i})\boldsymbol{tan(\emptyset'_i)}]}{\sum_{i=1}^{n}(\boldsymbol{W_i} + \boldsymbol{P} * \boldsymbol{b})\boldsymbol{sin(\alpha_i)}} \quad .16$$

2.4.1 Derivation of failure surface in terms of (x_0, y_0 ,R)

The fitness function is the definition of the factor of safety (FOS) in order to use the (MPGA) method to find the critical failure surface based on the Multi-

Parametric Convex Crossover (MPCX). The next step is to build the objective function.The first step is to identify where the slope boundary and the circular path intersect. The width of the slices is then determined by calculating the vertical distance between the top and bottom of the circular failure surface. Lastly, the point of intersection between the slice's midpoint and the slip surface boundary is calculated to determine the base angle (α) of the slices. Let the coordinates of the sites where the failure slip surface and the midpoints of the n slices, arranged from the toe to the crest of the slope, intersect be (x_1,y_1), $(x_2,y_2),\ldots,(x_n,y_n)$. Suppose that the x-coordinates of the crossing points at the slope's toe (X_L) and crest (X_U) are represented by these variables [27].The failure circle's equation is:

$$(x - x_0)^2 + (y - y_0)^2 = R^2 \quad .17$$

The formula for the slope's base is y = 0 and
The equation for the top of the slope is:

$$X_L = x_0 + \sqrt{(R^2 - y_0{}^2)} \quad .18$$

$$X_u = x_0 + \sqrt{(R^2 - (H - y_0)^2)} \;\; if\ y = H\ and\ x = X_u \quad .19$$

The width of each slices is :

$$b_i = \left(\frac{X_u - X_L}{n}\right) \quad .20$$

Where: n = number of slices.

For additional analysis, the angle formed by the circular slip surface's tangents at the interfaces of each slice is taken into account.

$$\alpha_i = tan^{-1}\left[\frac{x_0 - X_{Mi}}{Y_{Mi} - y_0}\right] \quad .21$$

The coordinates (X_{Mi}, Y_{Mi}) denote the places where the circular slip surface intersects the centerline of each slice, and the number of n nodes at each point determines the sliding surface's resolution. Tracing the sliding surface from the slope's toe to its crest yields these points.

$$X_{Mi} = x_0 + \sqrt{\left(R^2 + y^2{}_0\right) + bii} \quad .22$$

$$Y_{Mi} = y_0 + \sqrt{\left\{R^2 - (\sqrt{(R^2 - y_0^2) + b(ii + 0.5)}\right\}} \quad .23$$

The following is the equation for a slice's centerline:

$$x = \frac{(x_i + x_{i+1})}{2} \quad .24$$

$$x = x_0 + \sqrt{\left(R^2 - y^2{}_0\right) + b(ii + 0.5)} \quad .25$$

The angle of inclination of the slope "**β**": The total angle "β" is the angle formed by the two lines of combination, which are the points where the failure circle connects to the slope's base and peak.

$$tan(\beta) = \frac{M_1 - M_1}{1 + M_1 M_2} \quad .26$$

Where:

$$M_1 = \frac{y_0}{x_0 - X_L} \quad .27$$

$$M_2 = \frac{y_0 - H}{x_0 - X_u} \quad .28$$

The weight of i^{th} slice:

$$W = \gamma h_i b_i \quad .29$$

The height of the slope for i^{th} slice:

$$h_i = \frac{\beta}{10} R \quad .30$$

The length of the i^{th} slice:

$$l_i = \frac{b_i}{cos(\alpha_i\ )} \quad .31$$

Tangential force at each slice :

$$T = W sin(\alpha) = \ \gamma bh\, sin(\alpha) \quad .32$$

Normal force at each slice :

$$N = W cos(\alpha) = \ \gamma bh\, cos(\alpha) \quad .33$$

3 Working principle and operators of the proposed MGA Method

Natural genetics serves as the inspiration for genetic algorithms (GAs), which are mathematical models that mimic nature's capacity to generate, encourage, improve, and remove solutions to challenging optimisation issues. It is a search method for locating approximate answers to optimisation and search problems that was developed by [60], Back [61], Michalewics [62], Coello [63], Hedberg [64], Goldberg et al. [65], Zolfaghari et al. [20], Yang et al. [66], and Nian [67]. A subclass of evolutionary algorithms (EAs) known as genetic algorithms (GA) use methods like inheritance, mutation, crossover, and natural selection that are influenced by evolutionary biology. A population of abstract representations (referred to as chromosomes) of workable solutions (referred to as individuals) that evolve towards better solutions to an optimization problem are commonly used in the implementation of genetic algorithms. A population of utterly random individuals is the starting point for the evolution, which continues over many generations.

Every generation, the general fitness of the population is evaluated. Based on their fitness, serval individuals are randomly chosen from the current population, and they altered by recombination or mutation to emerge from a new population, which becomes the current one the next iteration. A list of parameters known as a chromosome, which is usually expressed as a straightforward string of data or instructions, represents a solution to a problem. Initially, a number of these people are selected at random from the original population. Every generation, each person is assessed, and a fitness function assigns a matching fitness value. The population is then sorted, with better solutions to the problem being reflected in those who are more suitable. One pair of parent organisms is chosen for breeding for every person generated in the following generation. The selection process is biased toward individuals from the initial generation with higher fitness; however, it is not exclusive that less fit individuals are entirely excluded. By doing this, the population is kept from prematurely convergently towards a local or poor solution. There are several well-known selection techniques for choosing parent organisms, including tournament and roulette wheel selection. After selection, the selected chromosomes undergo the crossover process, which recombines the organisms based on a predetermined probability.

Two new kid chromosomes are created by the crossover procedure and added to the following generation. The genetic material of the parent chromosomes is fused during this procedure, usually by switching out portions of their underlying DNA. Through this process, advantageous features from the parents are preserved while new variations are introduced into the population. Until the required number of candidate solutions are produced for the following generation, the crossover action is repeated using various pairs of organisms. The newly produced offspring must then be mutated. Mutation happens with a fixed and extremely low probability in a conventional genetic algorithm, typically 0.01 or less. Random alterations are made to a young organism's chromosome based on this chance. In the end, these genetic processes produce a new generation of chromosomes that is distinct from the original population. Through the introduction of novel features that would not be found in the current gene pool, mutation helps preserve genetic diversity within the population and avoids premature convergence. Mutation, in conjunction with crossover and selection, helps create a new population of chromosomes that is distinct from the original generation. Because only the fattest organisms from the first generation are chosen for reproduction, this process typically results in a rise in the population's average fitness.

This generational cycle is repeated until a termination criterion is met, such as reaching a maximum number of generations, hitting a minimal fitness threshold, or not observing any appreciable improvement over successive iterations.

3.1 Encoding

The effectiveness of the method is highly influenced by the way the coding is implemented. Coding refers to the representation of a solution (individual) as a string or vector, which can consist of integers, real-coded numbers, binary digits, or even alphabetic letters. In this study, the problem's search space is continuous. Hence, a real-coded modified genetic algorithm (RCMGA) is proposed. In continuous domains, the genes are represented as real codes (RC) for addressing parameter optimization problems. Therefore, a chromosome is a vector of floating-point numbers, with the precision depending on the machine running the algorithm. The dimensions of the chromosome are kept equal to the length of the vector representing the solution to the problem. In this way, each gene corresponds to a variable of the problem. Effectively, for many authors, such as Davis [68], in optimization problems where the variables have a continuous rather than discrete distribution in the solution space, representing the solutions with real-coded encoding is clearly more realistic and straightforward than using binary encoding. This method of representation also simplifies the implementation of the operators. For our optimization problem, the slip surfaces (solutions) are assumed two-dimensional arcs. Each circular slip surface is fully defined by three parameters: the radius and the two coordinates of its center. Therefore, the chromosome corresponding to a potential solution consists of three genes, forming a vector of three real numbers. This approach ensures that the chromosome, composed of meaningful genes, does not require decoding or repair.

Thus, a chromosome Ch_i of a potential slip surface is expressed by:

$$Chi = [xi, yi, Ri] \quad .34$$

3.2 Fitness function

The effectiveness with which each chromosome fixes the issue is then used to calculate a fitness value. The quality of the created solutions is measured by the fitness function. The chromosome serves as a failure surface in the research of slope stability, and the individual's fitness value is established by its factor of safety. Even if a chromosome has a very high factor of safety, it will be eliminated from the population if it generates a failure surface that cannot be examined. Although it is easier to follow the paradigm where the lowest value is optimal in all situations, it is possible to achieve a high fitness value by reversing the safety factor. According to the Fellenius estimate, the fitness function is selected as the factor of safety [03].

3.3Generation of the initial population

The first step in the optimization process is to generate a set of preliminary, workable solutions, which make up the initial population. This population consists of **NPop** randomly generated individuals, each representing a possible circular failure surface. To guarantee adequate diversity, the population size must be sufficiently large, allowing for the identification of the optimum solution. Generating an individual involves determining its three parameters within predefined intervals, as mentioned earlier. The analyst based on reasonable conditions and the slope boundary typically defines these intervals. That's to say:

$$xmin \le Xi \le xmax \; ; y\,min \le Yi \le ymax \qquad .35$$
$$Rmin \le Ri \le Rmax$$

Figure 5 displays the outcome of this stage as an example of the initial population formation. Following the creation of the initial population, each person's fitness is assessed. The following actions are then carried out in response to these assessments.

3.4Crossover process

By changing the three variables in a chromosome, one can change the location features of a slip surface, perhaps creating a new slip surface with a lower factor of safety. This modification is achieved through crossover and mutation operators. The crossover process begins by selecting pairs of parents from the current population to create a list of candidate parents [29].The size of this list:

$$N\_list = int((N\_Pop * T\_cx)/2) \qquad .36$$

Where:$int()$ is the integer function (N_{Pop}: population size, T_{cx}: crossover rate).

The more fit a chromosome is, the more likely it is to produce offspring, since the likelihood of an individual being chosen is directly correlated with its fitness. For each individual, the selection probability is calculated using a rank-based method. As previously stated, the suggested algorithm's operators have been changed to improve search effectiveness. To accomplish this enhancement, a novel Multi-Parametric Convex Crossover (MPCX) is used.By adding a local search mechanism, the suggested crossover operator stands out. By altering the convex combination parameter λ, this manifests in the creation of many child chromosomes. After that, Crossover takes the subsequent actions:

3.5 Mutation process

This operator plays a secondary role by preserving diversity within the population pool and helping the search escape local optima. It involves introducing minimal changes to some individuals' chromosomes. Mutation is applied at a small rate, affecting just a tiny percentage of the populace. The goal of this process is to displace a slip surface by perturbing the genes of a chromosome. First, some individuals are chosen at random from the existing population [29]. Then, for each selected individual, a simple uniform mutation is applied as follows:

$$Ch'[i] = Ch[i] for\ i = 1\ to\ 3 \qquad .37$$

4 Numerical experiments

This section applies several friction angles documented in the literature to solve a numerical benchmark problem in order to assess the validity and effectiveness of the proposed method. The technique uses the Fellenius method as the fitness function for these issues, which require homogeneous soil slopes. The grid search (GS) approach is first used to estimate the factor of safety (FOS), in which the center of each possible failure surface is limited to a predetermined rectangular region. Sound engineering judgment is used to identify this area's location. Within this grid, every point varies within a given range and corresponds to the center of a circular slip surface (c). Lastly, to determine the lowest value for the entire search region, the factor of safety is computed at each grid point [69, 70]. Similarly, the Multi-Parametric Genetic Algorithm (MPGA) is used to validate the methodology. Each member of the population is represented by a real coded chromosome in this method, which is matrixed with its variable (x, y) and R.

$$Mpop = \begin{pmatrix} x1 & y1 & R1 \\ x2 & y2 & R2 \\ xi & yi & Ri \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ xn & yn & Rn \end{pmatrix} \quad .38$$

Within the search space, every chromosome in the population represented a potential solution. To minimize the fitness function, the selected chromosome is assessed and evolves throughout the course of succeeding generations. By applying crossover and mutation operations, the performance of the algorithm is progressively improved. The final step in implementing techniques in the literature by Aniruddha et al. [27]. The critical failure surface (CFS) linked to the optimal factor of safety (FOS) is found in the stage using the (MPGA) algorithm. In comparison to conventional techniques, the findings indicate that the MPGA algorithm effectively determines the lowest factor of safety (min FOS), exhibiting faster convergence and better performance.

4.1case 1

The study conducted by Yamagami and Ueta [71] is the source of the geometryof the homogeneous soil slope depicted in Fig.8. Slope height H = 10 m, cohesiveness c′ = 9.8 kPa, friction angle ϕ = 10°, unit weight γ = 17.64 kN/m3, and pore pressure ru = 0 are the soil's geotechnical characteristicsshows in fig 8. Different numbers of slices are employed for the slope stability study; in this instance, n=20 slices are used. R is considered a random variable, as are the boundary variables (x_0, y_0). With load intensity q varying from 0 to 1000 KN/m2, the number of slices varied from n=4 to 20. The load width was also adjusted according to the number of slices. The performance of the critical failure surfaces (CFS) obtained using the genetic algorithm (GA) based on the Multi-Parametric Convex Crossover (MPCX) is displayed in Figure 9. The algorithm's tuning settings, which were meticulously identified as the optimal configuration for modifying the results, are shown in Table 2. These parameters were adjusted by varying the associated values within fixed intervals(Npop $\in [100-1000]$) and (Mpop∈[100-1000]),($T\_Cx \in [0.5{:}\,0.05{:}\,0.8]$) and mutation as ($T_mu \in [0.001{:}\,0.0001{:}\,0.002]$to get the best objective function. Table 3 displays the minimal Factor of Safety (FOS) values that were obtained from the slope analysis. The FOS values reported by other researchers are also included in Table 3 for comparison. The MPGA technique yielded crucial slip surface values for the minimum factor of safety (min FOS) that closely match the previously determined values, according to the comparative results of the current method and earlier research, which are compiled in table 3.

4.2case 2

According to the study, Fredlund and Krahn [09] use the geometry of the uniform soil slope depicted in Fig. 10. Slope height H=10 m, cohesiveness c′=29 kPa, friction angle ϕ=20°, unit weight γ=18.85 kN/m3, and pore pressure ru=0 are the soil's geotechnical characteristics. Different numbers of slices are employed for the slope stability study; in this instance, n=20 slices are used. R is considered a random variable, as are the boundary variables (x_0, y_0). With load intensity q varying from 0 to 1000 KN/m2, the number of slices varied from n=4 to 20. The load width was also adjusted according to the number of slices. Figure 11 shows how well the Multi-Parametric Convex Crossover (MPCX), a proposed genetic method, predicts the critical failure surfaces' (CFS) effectiveness. The results are shown in the algorithm tuning parameters, which were meticulously identified as the ideal configuration for fine-tuning Table 2. These parameters were adjusted by varying the associated values within fixed intervals (Npop $\in [100-1000]$) and(Mpop $\in [100-1000]$),($T\_Cx \in [0.5{:}\,0.05{:}\,0.8]$) and mutation as ($T\_mu \in [0.001{:}\,0.0001{:}\,0.002]$ to get the best objective function. Table 4 shows the minimal Factor of Safety (FOS) values that were obtained from the slope analysis. The (FOS) values reported by other studies are also included in Table 4 for comparison. The minimum safety factor (FOS)-related essential slip surface obtained using the (MPFA) method closely resembles the previously calculated values, according to Table 4, which highlights the comparative results between the current methodology and past research. As a result, all required terms have been computed. One may calculate the factor of safety (FOS) associated with a specific failure surface by replacing each slice in eq. 14 with these terms. The pseudocode for determining the slope's factor of safety is shown in Algorithm A.

Table 1 *Presentation of ranges associated to variables*

Applications	Variables	Range	Number of slices
1	$[x_0, y_0, R]^T$	$[(x,\ x+20), (H,\ H+10), (0,\ 50)]^T$	20

$*\, x = H * cot\ (\beta)$ $*\, n$: *totalnumberofslicesused*,$*\, b$: *wideofeach slice*, γ_w: Unit weight of water = 10kN/m^3

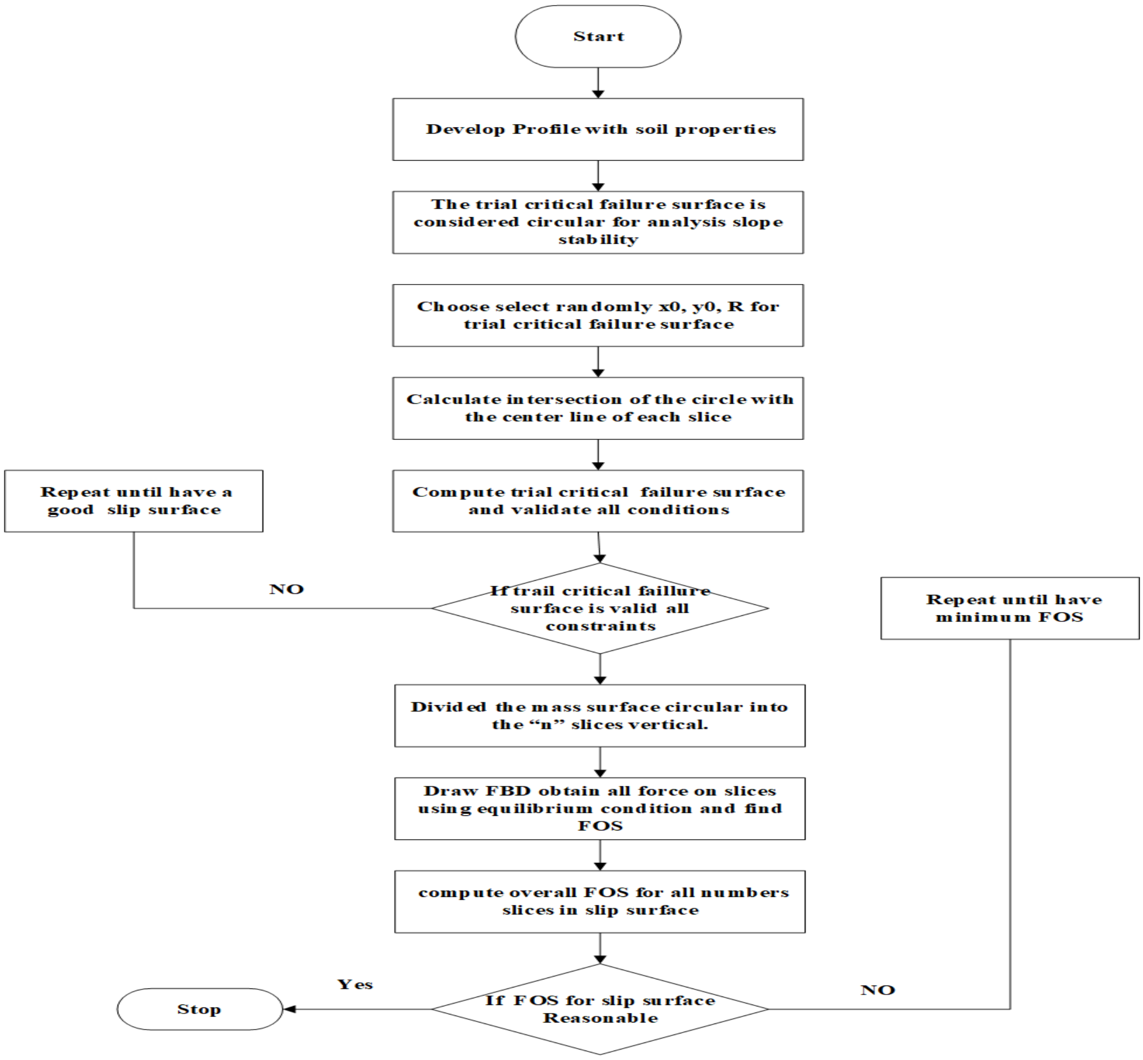

Figure 1. *Slope stability analysis workflow diagram.*

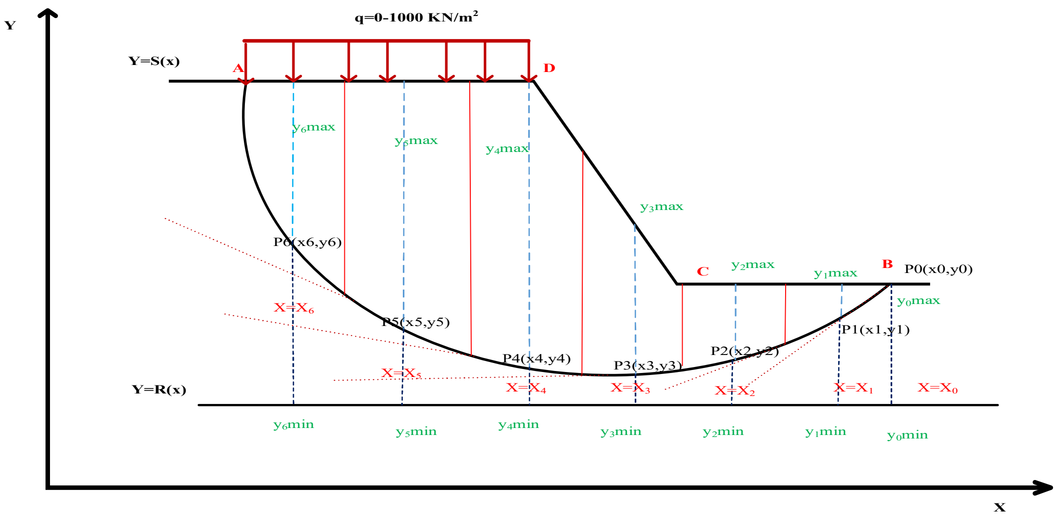

Figure 2. *The control variables that define the admissible slip surface are*

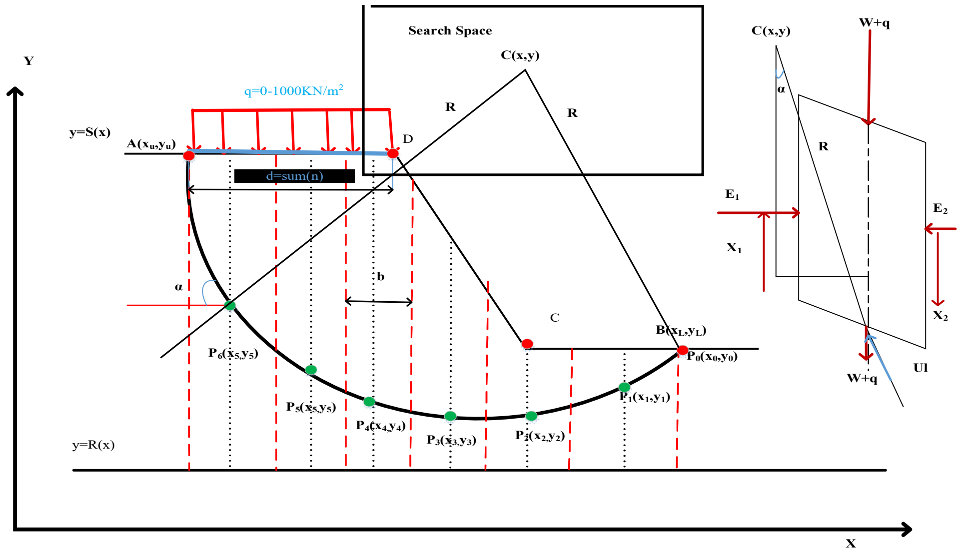

Figure 3. *Slip surface comprising finite vertical slices*

Algorithm 1: pseudocode for deriving factor of safety (FOS)of slip

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Inputs :

1. Constant parameters : Cohesion (C'); Friction angle (Φ'); Unit Weight(W); surcharge intensity(P)
2. Varying parameters : Centre points(x0,y0) and radius (R)

Result : Factor of safety (FOS) for failure surface

Step 1: Initialize slope surface geometry.
Step 2: Derive the slip surface in the terms (x_0,y_0) and R
while slip surface =valide surface **do**
  2.1 Randomaly select (x_0,y_0) and R for slip surface
  2.2Find intersection points (x_L,y_L) and (x_U,y_U) between slope boundary surface and slip circle
  2.3 Divide the slope mass into n finite slices
  2.4 Derive all n points [$P_1(x_1,y_1)$, $P_2(x_2,y_2)$, …, $P_n(x_n,y_n)$] that define in n-2 slices of failure slip surface
  end

Step 3:
  for i=0 : n-2 tel que n-2 number of slices
  3.1 Derive width (b) , Angle of slice base (α), Tangential force of slice, Normal force on the slice
  3.2 Calculate (FOS) for i^{th} slice
end

Step 4 :Stop

Figure 4. *Pseudo code for deriving factor of safety (FOS)*

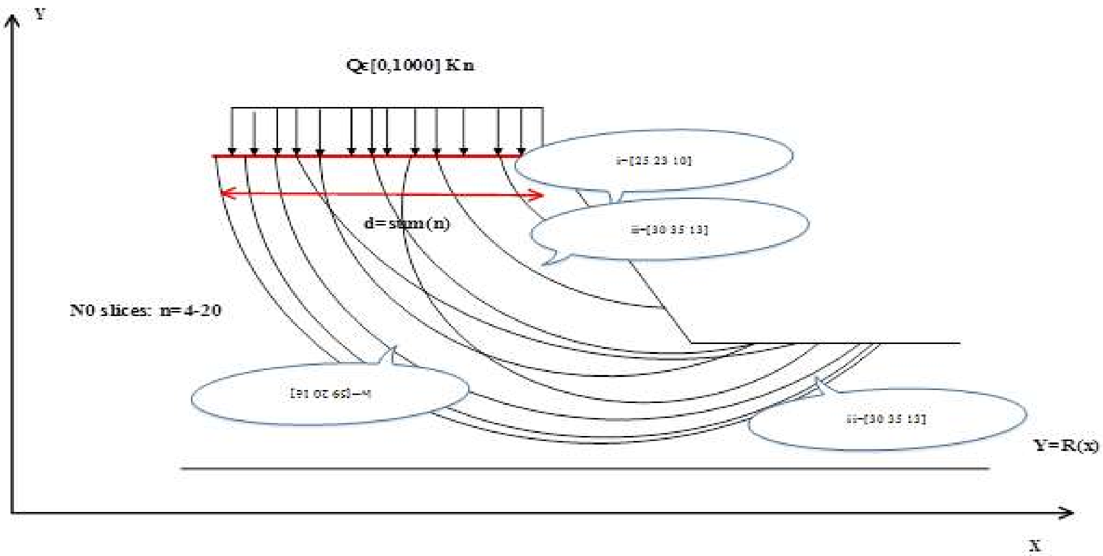

Figure 5. *An illustrative example of randomly generated initial population* $(NPop = 100)$

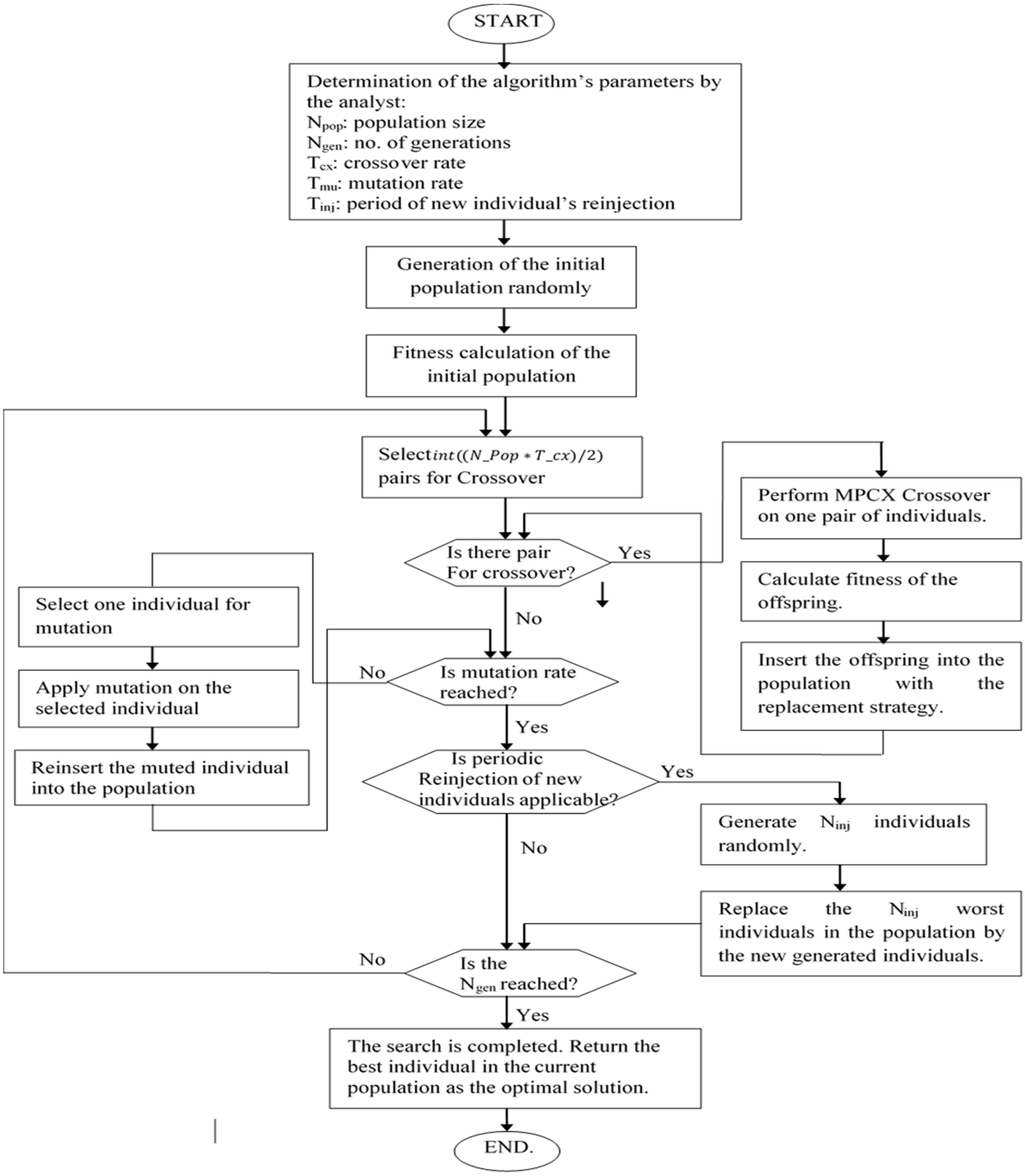

Figure 6. *The suggested Multi-Parametric Genetic Algorithm flowchart.*

Pseudocode for Crossover operations in MPGA

Step 1: For i = 1 to N_{cx}; take a pair of chromosomes from the crossover selected list, these chromosomes are known as parents; that say P_1 and P_2;

Step 2: generate at random a real number λ between [0 1] with uniform distribution;

Step :3 For i = 1 to 3, Calculate the tow children :

Ch[i]=λ*P1[i]+(1-λ)*P2 and

Ch[i+1]=λ*P2[i]+(1-λ)*P1

Step 4: then go to **Step** 2;

Step :5 From, return the two best children chromosomes as offspring of the tow parents.

Figure 7. *Pseudocode for Crossover operations in MPG*

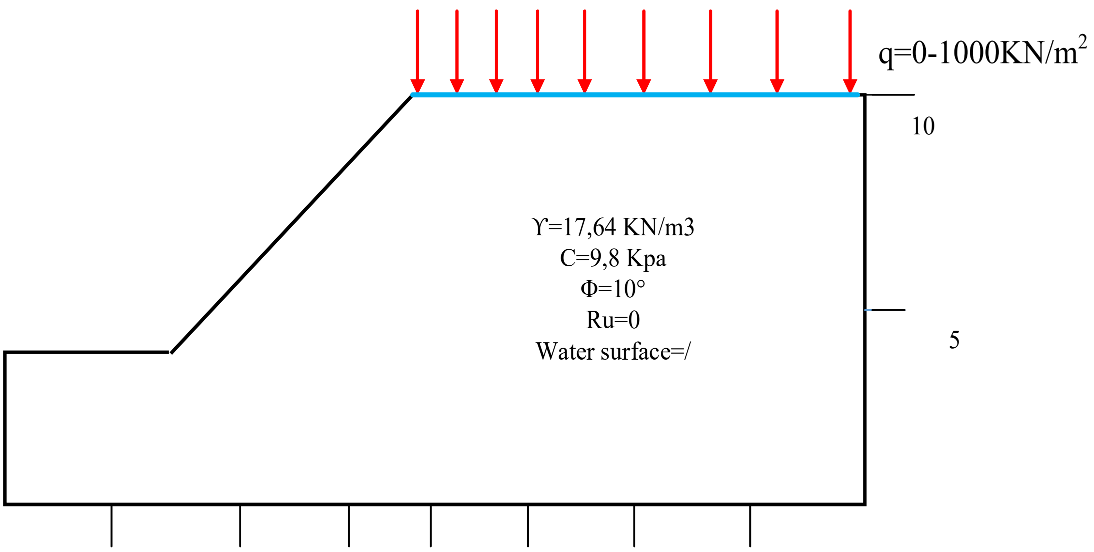

Figure 8. *The geometry and geotechnical properties of the slope model used in case study 1*

Table 2. *Parameters of the MPGA*

Population size	1000
Iterations	1000
Mutation Probability	0.002
Crossover probability	0.75
Crossover	Tow point

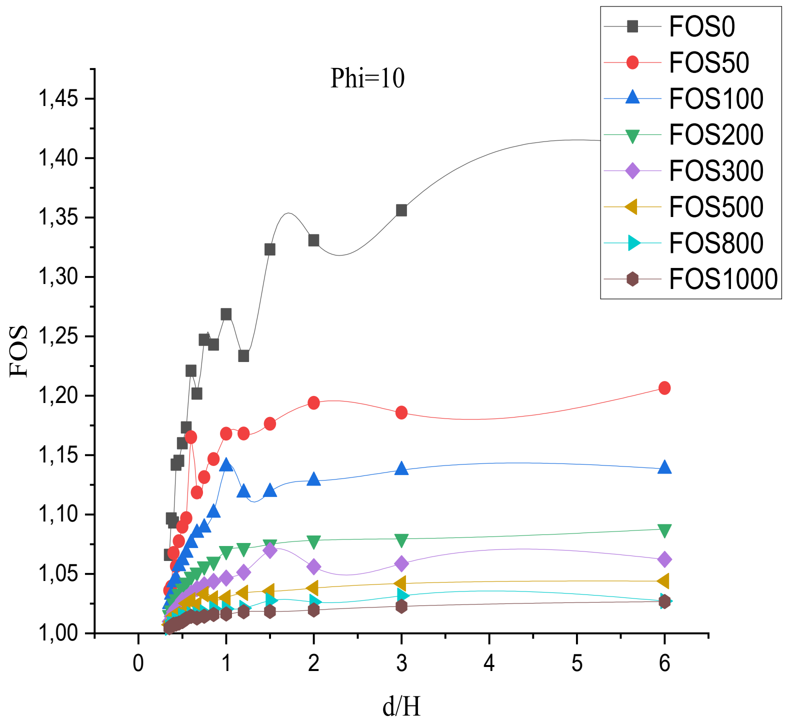

Figure 9. *Performance of the proposed MPGA for estimation of (FOS) with different loads applied for case study 1*

Table 3. *Comparative of analysis factor of safety summary results for case study 1*

Sources	Optimization Method	Minimum FOS
Yamagami and Ueta [71]	Broyden Fletcher Goldfarb Shanno (BFGS)	1,338
Yamagami and Ueta [71]	Simlpex Method	1,339
Cheng et al. [50]	Particle Swarm Optimization (PSO)	1,329
Cheng et al. [50]	Modified Particle Swarm Optimization (MPSO)	1,326
Cheng et al. [72]	Modified Harmony Search (MHS)	1,322

Jianping et al [73]	Genetic Algorithm (GA)+line	1,324
Jianping et al [73]	Genetic Algorithm (GA)+Spline	1,321
Kahatadeniya et al. [74]	Ant Colony Optimization (ACO)	1,311-2,966
Khajehzadeh et al. [75]	Modified Particle Swarm Optimization (MPSO)	1,308
Kang et al.[76]	Artificial bee colony optimization (ABO)	1,321
Jayraj Singh [36]	Grid search method	1,237
Jayraj Singh [36]	Biogeography based optimization	1,224
Present study	MPGA (P=0)	1,066-1,221
Present study	MPGA (P=50)	1,036-1,163
Present study	MPGA (P=100)	1,024-1,076
Present study	MPGA (P=200)	1,015-1,047
Present study	MPGA (P=300)	1,01-1,033
Present study	MPGA (P=500)	1,0072-1,027
Present study	MPGA (P=800)	1,0047-1,016
Present study	MPGA (P=1000)	1,0047-1,014

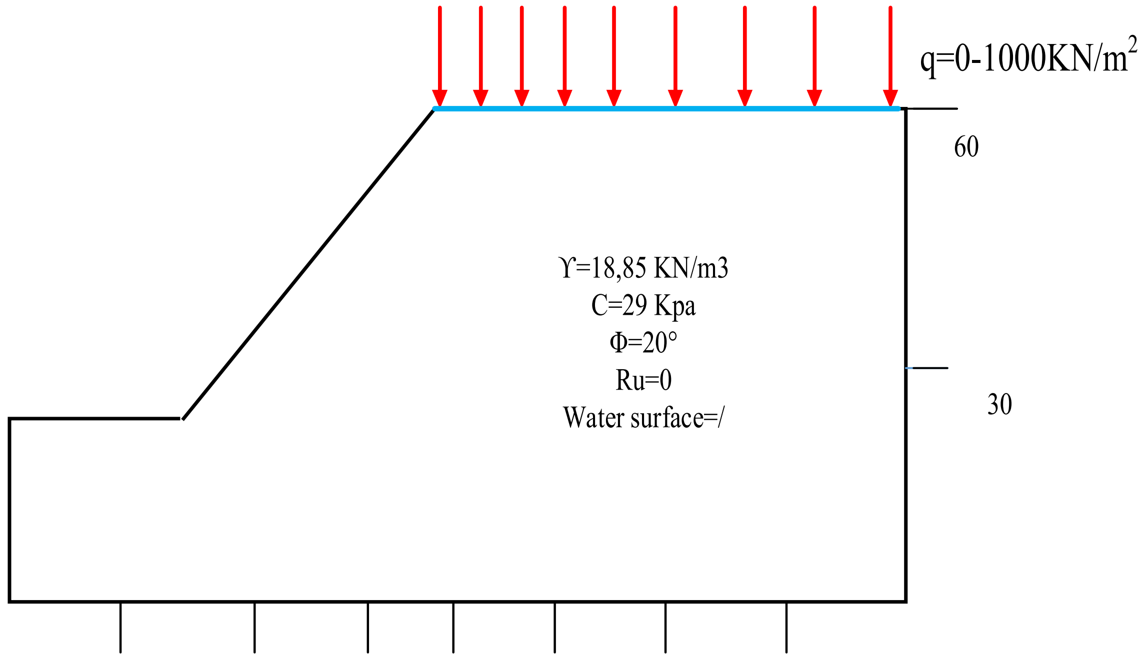

Figure 10. *The slope model utilized geometry and geotechnical characteristics in case study 2*

Table 4. *Comparative overview of case study 2 of the factor of safety findings*

Sources	Optimization Method	Minimum FOS
Fredlund and Krahn [09]	Fellenius method	1,928
Fredlund and Krahn [09]	Simplified Bishop method	2,08
Fredlund and Krahn [09]	Spencer method	2,073
Fredlund and Krahn [09]	Janbu Simplified method	2,041
Fredlund and Krahn [09]	Janbu Rigorous method	2,008
Fredlund and Krahn [09]	MP method	2,076
Baker .R [1]	Spencer method	1,98
Jayraj Singh [36]	Grid search method	1,923
Present study	MPGA (P=0)	1,074-1,36
Present study	MPGA (P=50)	1,036-1,213
Present study	MPGA (P=100)	1,025-1,13
Present study	MPGA (P=200)	1,015-1,10
Present study	MPGA (P=300)	1,011-1,065
Present study	MPGA (P=500)	1,0076-1,048
Present study	MPGA (P=800)	1,0053-1,029
Present study	MPGA (P=1000)	1,0039-1,021

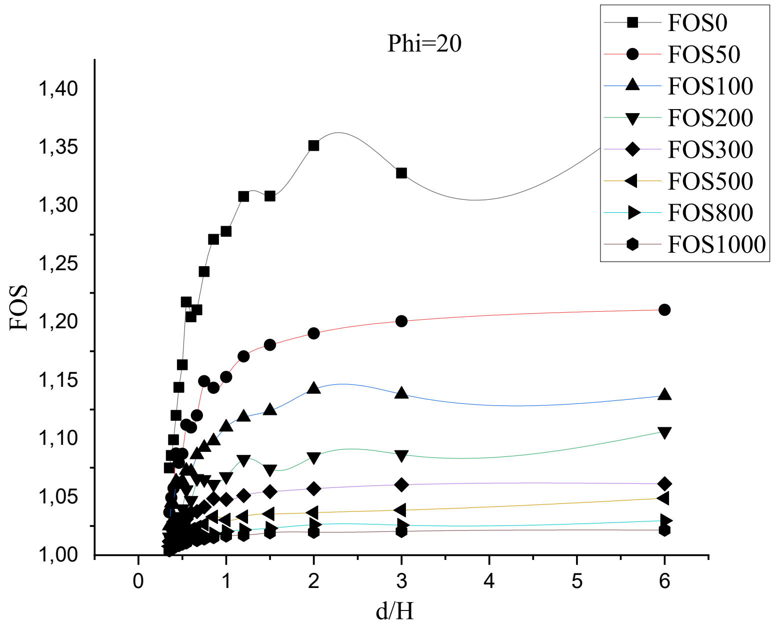

Figure 11. *Performance of the proposed MPGA for estimation of FOS with different loads applied for case study 2*

5. Conclusions

This research employed the Multi-Parametric Genetic Algorithmic (MPGA) to solve a nonlinear geotechnical problem by finding the critical failure surface (CFS) and evaluating the factor of safety (FOS). Using the Felleunis Method and Multi-Parametric Convex Crossover (MPCX), a computer program was created to determine the critical failure surface (CFS) and estimated factor of safety (FOS). In comparison to other crossover operators, the results indicate that the MPCX operator improves the real-coding modified Genetic Algorithm's (RCMGA) performance. As a point of comparison, the MPGA is effectively used to deal with the slope stability problem in an example where the friction angle varies between 10° and 20°. According to the case study, the algorithm performs favorably to other evolutionary algorithms that have previously encountered similar circumstances and yields excellent results, making it robust. Compared to other optimization techniques, the MPGA requires less or equivalent fitness function evaluations. The present work shows that the fitness function successfully converges to the global minimum and requires fewer parameters to be changed than some other bio-inspired algorithms. The MPGA's comparatively straightforward mathematical structure—which only requires the crossover and mutation of two parameters—is one of its main advantages over other well-known algorithms. Consequently, the proposed method may explore solutions utilizing a variety of control variable combinations and has greater search capabilities than (GS).

The (MPGA) can escape local minima more successfully than its GS equivalent thanks to this technique. According to the results of numerical experiments, the solutions generated by MPGA and BBO have lower standard deviations than those generated by other optimization methods. This implies that researchers and practitioners can use the current study's findings to choose the best metaheuristic approaches for their particular applications. The recommended method disregards Pore water pressure's impact because all case studies are evaluated at fry conditions, or zero pore water pressure. Furthermore, there is no discussion of how to assess slope stability in seismic circumstances. The quantity of slices ranges from four to twenty; we investigated in this article the impact of various stresses on the breadth slope. We saw how the safety factor (FOS) was impacted by the quantity of slices' n' in each load scenario.

Author Statements:

- **Ethical approval:** The conducted research is not related to either human or animal use.
- **Conflict of interest:** The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper
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- **Funding information:** The authors declare that there is no funding to be acknowledged.

- **Data availability statement:** The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

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