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Research Article

Patient Routing for the Emergency Department: A Simulation-Optimization Framework for Reducing Travel and Service Times

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Abstract:

Emergency department overcrowding poses a significant challenge in healthcare systems worldwide, creating significant inconveniences for both patients and staff. Although numerous studies have attempted to address this issue through system-level solutions within emergency departments, the management of patient flow prior to reaching and overcrowding hospitals remains unaddressed. Currently, the best practice involves diverting arriving ambulances to alternative hospitals when an emergency department is overcrowded, which introduces additional delays in patient treatment and care. Such chaotic circumstances in the existing process highlight the need for a solution that organizes patient and ambulance flow such that regional emergency departments share the workload fairly, based on their capacities and capabilities, while minimizing patient travel time to the most convenient and suitable emergency unit on the first attempt, ultimately reducing treatment and care time significantly. In this regard, this paper proposes a novel approach to direct patients and ambulances—before heading to a hospital—to the best and most convenient emergency department. This is achieved by broadcasting the status of emergency departments in the region on an hourly basis to the public through a mobile application. The broadcast policy, spanning the hours of the day, is derived using a simulation-optimization model based on travel times, patient demand, and emergency department process durations analyzed using real data. This study is unique in pioneering emergency patient flow management outside of hospitals to maximize patient benefits and enhance service quality broadly. The simulation-optimization model is applied in a five-district region and three hospitals with emergency departments. The optimal hourly broadcast policy achieved an 11% reduction in service time, a 9% reduction in laboratory time, a 41% decrease in radiology time, and a 26% reduction in consultation times, which are significant in terms of human health.

1. Introduction

Due to the increasing prevalence and negative impact of diseases (e.g., pandemics) in health, social, and economic life of people, it has become crucial for countries to invest more in healthcare infrastructure and services and prioritize this field in government's investment list. However, it is also known and expected that the budgetary resources required for such investments are relatively high and may not be met easily. This poses a significant challenge, especially in low-income countries. In the bottom 60 countries in terms of the income ranking, healthcare expenditures per capita reach only $15 annually, while developed countries' annual expenditures can go as high as $2000 [1]. For example in Türkiye, where a study is conducted and will be presented in this article, healthcare spending per capita reached $1064 in 2015, while the average for OECD countries was $3813 in the same year [2].

The rising healthcare costs and the increasing shortage of high quality healthcare services due to inadequate investments in upgrading infrastructure capabilities and capacities [3], highlight the need for studies on healthcare operational efficiency. In this context, emergency departments (ED) are one of the most comprehensive units in the healthcare sector, providing 24/7 service. They are often overcrowded and busy for being the first point of contact when sudden and urgent health issues arise particularly in off-working hours [4]. Like many healthcare centers, emergency departments also experience overcrowding, which can lead to various irrecoverable problems. Insufficient numbers of stretchers, inadequate availability of emergency doctors or nurses, physical capacity limitations, and other resource inadequacies, as well as the continuous increase in emergency service demand, are just some main reasons behind the density observed in emergency departments [5]. Inefficiency [6] and lack of selection [7] in ambulance offloading operations, overcrowding in the departments where patients will be referred [8], the expectation of faster examination in the emergency department [9], the increasing severity of illnesses despite the lack of an adequate safety net of social support or insurance [10], increased admission rates in the emergency department, and changes in healthcare policies, such as emergency service billing, are other factors contributing to the increasing intensity and unpleasing situations in emergency departments. Delays in patient transfers and misdirected ambulances, further exacerbate complications in emergency department [11]. In this regard, among many, it is worth mentioning some of daily crises arising in an emergency department as follows: On the patient side; prolonged triage and waiting times result in patients leaving without receiving emergency medical care [12], reduced examination times due to high patient volumes [13], and combined with delayed treatments lead to a decrease in service quality that may create further serious health issues and even fatalities. On the emergency department side; the high workload of personnel limits sufficient rest periods, increased stress from dissatisfied patients lead to verbal and/or physical violence, resulting in personnel resignations [6]. Continuing on the emergency department side; it can be added that considering the human factor, fatigue, and overcrowded environment increase the likelihood of healthcare errors. In further addition, there is also a financial aspect to the situation; a study by Hoot and Aronsky (2008) [14] revealed that an additional day spent in the emergency department on average cost a hospital $6.8 million over a three-year period. In order to find a way out to avoid or minimize all above negative circumstances, Sayah et al. (2016) [15] conducted a study and compared the effects of capacity expansion with system improvements on patient flow in a hospital setting. The findings revealed that improvements such as establishing a rapid assessment unit and training staff as patient partners enhanced patient satisfaction, reduced waiting times, and decreased the number of patients who left without being seen. Consequently, it can be said that any enhancement of patient flow in an emergency line has a great potential to improve overall service quality in a hospital. However, most existing studies to find solutions for overcrowding problem have focused only on improving internal patient flow efficiency within the boundaries of a hospital, although one of the root causes of the problem is external, accepting emergency patients irregularly delivered to the hospital.

It is a fact that all these problems can be minimized if the problems are prevented from occurring at the beginning through directing a patient from his/her first location to the most convenient regional emergency department.

In this regard, the subject matter study aims to initiate research, development, and innovation studies in usually overlooked area of emergency overcrowding problem, which is managing patient flow outside of emergency departments.

The study yielded an effective decision-making model that can run on a mobile application accessible by ambulance staff to direct ambulances to the most convenient emergency department, by utilizing real data of travel times as well as capacity, capability and workload balance of available emergency departments.

From a finite set of levels indicating emergency departments' availability or crowdedness, a selection for public announcement of status is made for hour of the day is made for each emergency department. This information is broadcast to inform and direct the patient population in need of emergency services. Shared information is available for both walk-in patients and those arriving by ambulances, or their relatives that are in the position of decision making in the emergency condition. We seek to find the optimal daily broadcast policy covering the 24 hours of the day. This policy determines the level of availability/crowdedness announced for each emergency service and each hour of the day. Using simulation-based optimization, a policy that minimizes the average time spent in the system, including transportation and processes in the emergency department, from the onset of the patient's need for emergency care is determined and proposed to the patients and ambulances.

The following section of the study reviews relevant literature on simulation, optimization, and efficiency in emergency departments and healthcare services. In Section 3, the emergency department process flow adopted in the simulation model and input analysis of data from patients and travel durations is discussed. The section culminates in the presentation of the simulation-optimization model. In Section 4, the existing system operational performance and the simulation-optimization solution data are compared, and the improvements are reported. Section 5 presents conclusions and future research directions.

2. Literature Review

Simulation is a computational modeling technique used to estimate and evaluate the performance of a modeled system within certain conditions and over a specified duration, representing the workflow in a system [16]. Simulation models are particularly suitable for gaining insights into changes that are proposed for a system but would be costly or time-consuming to test directly. Simulation models are frequently utilized in many healthcare organizations for management [17], design, and performance evaluation [18].

A systematic review of the applications of simulation techniques in emergency departments revealed that out of 254 publications between 2000 and 2016, 224 focused on flow processes and system performance in emergency departments. While the majority of studies concentrated on resources such as physicians, nurses, and beds, a limited number of studies examined patient behavior [19]. In a study that improved resource planning for increased efficiency, a discrete event simulation pointed out a 48% reduction in waiting times [20]. Another study showed that efficient planning of patient beds had a greater impact on system performance than physical space and workforce resources [21].

Hussein et al. (2017) [22] used the National Emergency Department Overcrowding Scale (NEDOCS) as a measure to assess the level of crowding in emergency departments. NEDOCS is calculated based on parameters such as patient waiting time, the number of beds in the department, and the number of patients admitted to the hospital and emergency department. The authors aimed to reduce NEDOCS and patient waiting times through technological enhancements of medical devices and equipment used in the emergency department. In another study utilizing discrete event simulation, Yang et al. (2016) [23] investigated the optimal numbers of physicians, nurses, and laboratory capacity to maximize the performance of the emergency department. Simulation models have been used in studies focusing on resource requirements and bottleneck identification in the emergency service system [24], comparison of developed patient flow scenarios and different resource levels [25], determining the impact of resource utilization on hospital cycle times for emergency department patients [26], and determining the effect and prioritization of staff numbers for doctors, nurses, and registration personnel on cycle times [27]. Additionally, the impact of bed capacity on patient costs [28], balancing doctor waiting times and its effect on patient cycle times [29], and enhancing ambulance service speed and emergency system efficiency through efficient scheduling [30] are among the research focuses where simulation models have been utilized.

A systematic review conducted by Wiler et al. (2011) [31] classified studies on the problem of emergency department crowding and intensity based on the analytical techniques used. According to their findings, mathematical formula-based models, regression or time series-based models, queuing models, and discrete event simulation models are the most commonly employed analytical methods in studies addressing emergency department crowding/intensity issues.

Simulation-based optimization has been applied in various publications that focus on analyzing and solving problems related to crowding and intensity in emergency departments. Ahmed and Alkhamis (2009) [32] aimed to minimize patient waiting times by determining the optimal number of staff members using a simulation model they developed for an emergency department. Their study demonstrated a 40% reduction in waiting times in the simulation model, with a 28% increase in the number of staff members. Another study utilized discrete event simulation to optimize the length of stay in the emergency department and emphasized the importance of the time between arrival and initial examination. By incorporating a personnel budget constraint, they were able to minimize waiting times [33]. In a study that addressed nurse scheduling for improved service efficiency, simulation modeling and genetic algorithms were combined to reduce queue lengths and increase patient satisfaction [34].

While the majority of existing studies have focused on improving efficiency through changes in the internal workings of the emergency system, there are limited studies that explore alternative approaches. In a study by Laskowski and Mukhi (2009) [35], which was conducted during a period when mobile phones were just beginning to become widespread, they proposed an infrastructure based on ambulance redirection and utilized simulation-based optimization to design a system that integrates and publishes crowding data from emergency departments in a region. Although the study mentioned sharing information about the status of emergency departments publicly to prevent overcrowding, it mainly focused on the policy of patient or ambulance diversion (redirecting patients to other emergency departments due to overcrowding) which has been addressed in studies quantifying contributors [36] or discussing potential solutions [37]. In these diversion studies, informing and directing patients at the location where their demand arises or considering travel times is not typically discussed, and redirected patients may have to endure additional travel times while congested emergency departments close their doors to patient admissions.

In most studies that address overcrowding and intensity issues in emergency departments using simulation and simulation-based optimization, improvements have been focused on the internal workings and resource utilization within the emergency system. However, there are studies considering emergency departments in an area as a network, and pointing out the advantage of utilizing these facilities as a collective resource. Several key studies have paved the way for understanding and addressing overcrowding and intensity issues in EDs by employing network-focused strategies. For instance, Deo and Gurvich (2011) [38] delve into central decision-making on ambulance diversion to enhance the efficient utilization of a network of EDs. They employ a queuing game model between two EDs that aim to minimize their own waiting times, revealing that decentralized decision-making often leads to a defensive equilibrium where EDs do not accept diverted ambulances from each other, thereby exacerbating delays. Their study proposes an alternative coordination solution that approximates optimal network conditions without requiring precise knowledge of problem parameters, aiming for a more effective approach to managing ambulance diversion and improving overall ED efficiency. Ramirez-Nafarrate et al. (2012) [39] explore ambulance diversion policies through Markov Decision Policies and simulation. They compare no diversion to an ambulance diversion policy applied when all ED beds are full, and point out that patient waiting times can be reduced by diversion policies derived using the Markov Decision Process model. Piermarini and Roma (2023) [40] take a broader approach by examining network-wide optimization and treating EDs within a network as collective resources under a simulation-optimization framework. They compare various ambulance redirection policies, ranging from no redirection with patient queuing to diversions based on resource availability, patient priority, or to the least occupied ED irrespective of distance. Their findings demonstrate potential in reducing waiting times and costs, yet they underscore that current ambulance diversion policies are often only enacted when EDs face extreme overcrowding and do not account for patients arriving by private means. These studies collectively highlight the benefits and challenges of considering EDs not just as individual entities but as parts of a larger, interconnected network. This shift in perspective is crucial for developing more comprehensive and effective strategies to manage patient flow and optimize emergency care services.

Building on previous research that often focuses on ambulance diversion under specific, often extreme conditions, the proposed intervention in this study offers a comprehensive solution that addresses the entirety of patient flow to EDs. Unlike traditional approaches that respond reactively at the ED gates, this study harnesses mobile technology to proactively manage patient distribution across the network. By deploying a mobile application that provides real-time information on service times and crowdedness levels at various EDs, this intervention guides patients to facilities where they can receive faster care, thereby optimizing the performance on the entire network. Simulation-based optimization is used to determine the optimal times to broadcast the status of each ED, aiming to balance overcrowding and underutilization dynamically throughout the day. This method considers not only ambulance traffic but also the movement of patients arriving by private means, a significant source of patient flow that previous studies have largely overlooked. By integrating traffic data with real-time updates on ED capacities, the system directs patients to less crowded EDs before they even begin their journey, significantly reducing overall system times, including both travel and wait times in the ED. This strategy not only minimizes waiting times but also alleviates crowding, thereby improving service delivery and enhancing satisfaction for both patients and healthcare staff. The use of widespread mobile technology and centralized decision-making to direct demand in real-time represents a pioneering contribution to the field of emergency department efficiency, pointing out and analytically investigating opportunities for how emergency medical services can be delivered across a network of facilities.

3. Methods

Individuals requiring emergency care—whether arriving by private transport or ambulance—are assumed to use a mobile emergency medicine application. This application delivers real-time, hourly updates on ED congestion, assisting patients or their decision-makers in selecting an ED based on its current availability. The route from the emergency location to the ED is fixed, while the travel pace adjusts based on traffic conditions. The total duration required for a patient to access emergency healthcare is estimated by considering both the travel time and the congestion within the ED. The system optimizes the decision regarding which availability levels to broadcast for each ED, thus enhancing ease of access by potentially guiding patients towards options that are both closer and less congested. The primary objective is to minimize the overall average time spent traveling and receiving services at the ED. A detailed simulation model of the ED workflow is constructed to ensure the accuracy of the optimization process, as elaborated in Subsection 3.1. Moreover, the analysis of the examination, consultation, laboratory testing, and radiology imaging processes, as well as the durations and travel times to the ED, are discussed in Subsection 3.2.

3.1 Workflow in the Emergency Department

The flow within the emergency department is characterized by the collection of vital information obtained through direct interviews with emergency service personnel. The primary objective of this study centers around the concept of "early access to care." Consequently, the analysis focuses on various processes that occur during the duration of emergency department services, including the initial examination, testing, diagnosis, re-examination, and consultation. However, it does not encompass procedures such as admission, prescription, transfer to another hospital, or referral to outpatient clinics. These latter procedures are either included within the aforementioned stages (admission being considered within the input analysis for the initial examination) or are administrative tasks that follow the completion of actual services, primarily associated with the discharge process.

The system initiates with the patient's arrival at the emergency department, where they enter the queue for the initial examination and await the attending emergency physician. During the initial examination, decisions regarding the patient's condition and the need for further testing and treatment are made. In cases where laboratory tests are not required but medication is deemed necessary, prescriptions are issued, and patients are discharged. Likewise, if medication is not deemed necessary, patients are discharged without the need for further intervention. If laboratory tests are requested by the physician, the emergency nurse collects the required samples for analysis, and radiology examinations are ordered. The emergency physician awaits the return of laboratory results and the availability of radiology examination results before reassessing the patient. Based on the initial examination and/or test results, the emergency physician may call upon specialist doctors from respective departments/clinics for consultation. The consulting doctor, who responds within a time frame depending on the workload of the respective department, collaborates with the emergency physician to evaluate the patient's condition. The completion of all testing, evaluation, and consultation processes is regarded as the culmination of the emergency service for the patient. Figure 1 depicts the emergency department flow as adopted in the simulation model utilized in this study. Note that laboratory, radiology, and consultation stages proceed in parallel; with the radiology requisitions by the physician, the patient is practically queued for radiology imaging in the hospital's information system. Similarly, the doctor may call a specialist for a consult during an initial examination, or a later re-examination, which is entered into the information system directly, starting the waiting time for the arrival of the specialist. Then, the only physical activity that might take several minutes is sample taking by the emergency department nurse, which does not interfere with consultation and only very minimally with the radiology process with a small probability.

3.2 Data Analysis

To conduct the input analysis for this study, data provided by the Information Technology Department of Ankara Numune Training and Research Hospital were utilized. Anonymized examination, testing, laboratory, consultation, and relevant time data of 164,579 patients who sought treatment at the hospital's emergency medicine unit between January 1 and October 31, 2018, were statistically analyzed. The data tables included records of patient entry into the emergency unit, consultation/diagnosis information, laboratory test requisitions and results, as well as radiology examination requisitions and result retrieval times. The data input for constructing the simulation model of the healthcare system involved analyzing various aspects such as inter-arrival times, durations of initial examination, tests, and consultation requests, along with the time intervals for their initiation/request, response, and/or result generation. ANOVA was used to determine if the distributions varied based on categories such as months of the year, days of the week, hours of the day, and the diagnosis of the patient (if available) or the type of laboratory test. After identifying the appropriate classification/grouping, distribution fitting was performed using the MATLAB statistical toolbox.

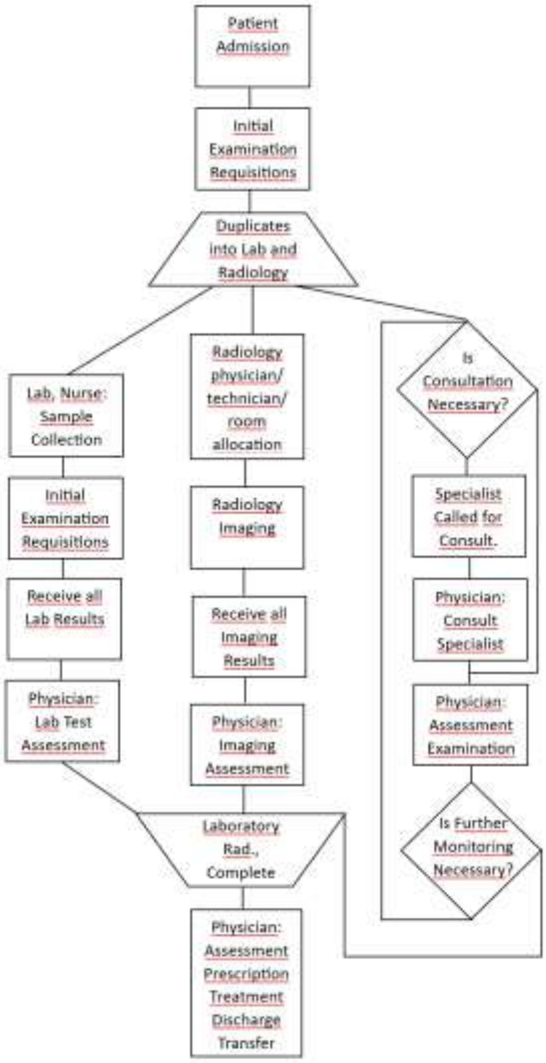

Figure 1. *Flow chart of the emergency department flow adopted in the simulation model.*

The fit of different distribution types, including discrete and continuous distributions, was compared using histogram-probability density/distribution function plots and quantile-quantile (Q-Q) plots. The most suitable distribution among exponential, gamma, Weibull, and log-normal distributions was selected based on these analyses. The parameters of the selected distributions were estimated using the maximum likelihood method, and the probability density functions were plotted and compared to the scaled histogram of the observed data. The distribution with the highest compatibility in the obtained histogram-pdf plots was visually selected, and the decision regarding the appropriate distribution was made accordingly. For a more comprehensive information on analysis, including data cleaning, processing, visualization, and interpretation, refer to the study by Demir (2019) [41]. Travel time data was sourced from the Google Maps website. The travel times between five different locations and three emergency service centers were collected and analyzed for each day of a month (Table 1). Inter-arrival times varied based on the day of the week and the time of day, while the seasonal effect was found to be insignificant [41]. The exponential distribution exhibited compatibility when examining the durations between arrivals on specific days of the week and at specific hours. Therefore, the arrival counts for each day of the week at each hour were modeled with a non-homogeneous Poisson process with varying rates. The exponential arrival intervals provided in Table 2 are for the total demand of all districts and are disaggregated to districts based on hypothetical population proportions. The population ratios of the districts to the total population of the five regions which generate the overall demand were assumed to be 5%, 30%, 15%, 25%, and 25%. The study evaluated the durations required for the ED physician to conduct an initial examination, the ED nurse to collect specimens/blood, the laboratory to return test results, radiology technicians to conduct imaging, the doctor to initiate a new consultation call, and consultant doctors to respond to calls, to determine if these times varied depending on the patient's diagnosis or the type of test conducted. Patient condition/diagnosis did not significantly affect the time interval between patient arrival at the emergency department and the initial examination, as indicated by a one-way ANOVA test (*$p=0.43$*). Histogram-pdf and Q-Q plots suggest that log-normal distributions fit best for modeling waiting times for the initial examination.

Figure 2 depicts inter-arrival times during the first day of the week at the emergency department. The demand rises just before working hours, remains steady until midnight, and drops through the late night/early morning.

For laboratory tests, eight laboratory categories were identified: Hematology, Biochemistry, External Lab, Blood Gases, Elisa, Blood Center, Hormones, and Culture. Each group contained different types of tests ("tubes"). Based on the ANOVA results, it was determined that the durations varied significantly for the test type within the Hematology and Blood Center groups, which led to modeling the durations specific to each tube within these categories. For the other laboratory categories, ANOVA indicated no significant differences among the test types, resulting in modeling the durations specific to the entire group. Detailed results for statistical tests between and within laboratory categories and the distributions selected for laboratory tests are presented in [41]. Multiple radiology requests can be made for a patient to achieve the correct diagnosis or treatment. Since there is no available data indicating a relationship between two consecutive tests conducted on the same patient, the tests were examined based on the duration of the request,

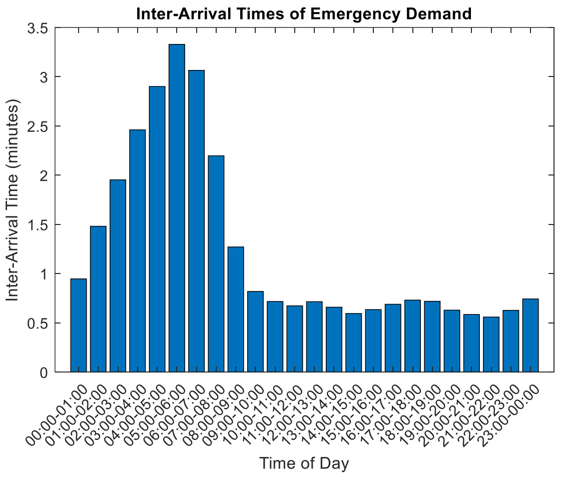

Figure 2. *Inter-arrival times during first day of the week to the emergency department analyzed.*

starting from the time of examination. ANOVA tests demonstrated significant differences in the number of tests requested ($p = 0$) and the durations of the requests ($p = 0.0117$) based on the patient condition/diagnosis. Yet, except for one case, the average differences between diagnostic pairs for all values were lower than the minimum significant difference (*MSD*) [41]. Due to the lack of significant pairwise difference, the radiology request durations and numbers were analyzed regardless of the diagnosis. The time required for radiology result retrieval was found to be independent of diagnosis by one-way ANOVA ($p = 0.58$). In certain cases, deemed necessary by the consulting emergency physician, consultations are conducted for some patients (approximately one-fourth of the patients) where the opinion of a specialist in a specific field is sought based on the judgment of the emergency physician. Calls are made to the specialist in the relevant field, and depending on the workload of the specialist in their respective department, they respond to the call and engage in consultation with the emergency physician upon arrival at the emergency department. For most diagnostic types, a single consultation is performed, but multiple consultations are required for a few rare diagnostic categories. In some diagnoses that undergo multiple consultations, the average number of consultations reaches ten, and in these categories, discharge with a single diagnosis is rare. Geometric distribution or generalized negative binomial distribution has been preferred in studies modeling the number of consultations, since there is a cycle in this process: the emergency physician conducts an examination, calls a physician for consultation if deemed necessary, follows up with a reexamination, and again decides whether or not to call for another consult. Since the decision to call a consultant is modeled as a Bernoulli process, the natural probabilistic model for the number of consultancy calls is geometric or negative binomial. One-way independent measures ANOVA test revealed a significant effect of the diagnostic category on the number of consultations ($p=0$), and the differences between pairs are mostly significant. Therefore, the distributions of consultation numbers were examined specific to the diagnostic category. The parameters of the negative binomial distribution are determined based on the maximum likelihood method, where the probability parameters are set to be equal to the minimum number of consultations in the diagnostic category. One-way independent measures ANOVA test demonstrated a significant difference in the response time of the consulting specialist to the call and arrival at the emergency department based on the diagnostic category ($p=0.025$). However, due to the rarity of differences larger than the minimum significant difference (*MSD*) when comparing the most common 20 diagnostic pairs, the response times of consulting specialists were modeled with a common distribution. Interarrival times for consultancy calls are similarly independent of patient diagnosis and fit an exponential distribution. Considering the existence of a weekly repeating pattern, the analysis

Table 1. *Minimum and maximum travel durations for hours of the day.*

	Monday		**Tuesday**		**Friday**		**Saturday**		**Sunday**	
Hour	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
0:00	14	20	14	18	14	18	14	20	14	20
1:00	14	18	12	16	12	18	14	18	14	18
2:00	12	16	12	16	12	16	12	16	12	16
3:00	12	16	12	16	12	16	12	16	12	16
4:00	14	16	12	16	12	16	14	16	12	16
5:00	12	16	12	16	12	16	12	16	12	16
6:00	12	16	12	18	12	16	12	16	12	16
7:00	16	24	16	24	16	24	12	18	12	18
8:00	18	35	18	35	18	30	14	20	12	18
9:00	16	30	18	35	16	30	14	22	12	20
10:00	14	24	18	30	16	28	14	22	14	20

encompassed all seven days of the week and all 24 hours of the day. The collected data provided the shortest and longest travel times between two locations. Weekday travel times differed from those on weekends and outside of office hours. For instance, an individual departing at 8:00 AM on a Monday experienced a minimum travel time of 18 minutes and a maximum of 35 minutes, while at the same time on a Sunday, the travel time ranged from 12 to 18 minutes. In the research reviewed on traffic durations, it is stated that the best traditional statistical distribution representing travel time is the lognormal distribution [42]. Rakha et al. (2006) [43] have shown through goodness-of-fit tests that the lognormal distribution better represents highway travel times. Emam and Al-Deek (2006) [44] compared four different travel time distributions (Weibull, exponential, lognormal, and normal), demonstrating that the most suitable statistical distribution is the lognormal distribution. Based on these views, and considering the difficulty of obtaining private vehicular travel times due to privacy reasons, we make the assumption that patient travel times at a certain hour of the day will follow lognormal distribution. Given that the available data consisted of the shortest and longest travel times, the travel times were modeled using a lognormal distribution to represent the range of travel times with a 95% probability. With the determination of appropriate duration distributions for each process, a discrete-event simulation model of the emergency department is constructed. This model incorporates various factors including the inter-arrival times of patients, the durations of processes such as initial examination, laboratory tests, radiology examinations, and consultations, as well as the staffing levels of doctors and nurses in each department. The simulation model facilitates the analysis of key performance measures, including waiting times, queue lengths, resource utilization, and patient flow patterns. However, the most significant aspect is its ability to calculate the mean total service time for patients in the emergency department. By simulating the ED using the developed model and considering different scenarios, we can assess the impact of routing emergency medicine demand to less crowded emergency departments. Furthermore, the simulation model enables the estimation of mean service times and travel times, thereby paving the way to the minimization of the overall system time for patients by a simulation-optimization approach.

3.3 Simulation-Optimization

The emergency system model presented in Figure 2 is constructed in the discrete event simulation software ARENA, 14.0. In analyzing the process flow and input analysis of an emergency department, the focus was on the core processes and staff, as the ED simulation model developed would eventually represent various EDs across different hospitals in multiple locations. The model captures only the emergency system dynamics, accounting for three distinct locations, each with varying staffing levels and different numbers and schedules of emergency doctors based on the size of the services. Table 2 details the number of emergency doctors per shift for each of these emergency services.

Table 2. *Schedules for number of doctors working in the shifts for the three emergency departments in the simulation-optimization model.*

	0-4:00	4-8	8-12	12-16	16-20	20-0:00
ED1	5	4	6	7	8	6
ED2	4	2	2	4	4	4
ED3	2	2	4	4	4	2

The incoming traffic is divided into five regions for analysis. Travel times from a designated point in a neighborhood (origin) to the location of an emergency service (destination) are provided as an example in Table 1. When no information is broadcast regarding the congestion level of emergencies, it is assumed that the probabilities of emergency requests being distributed to the emergency services from districts are presented in Table 3.

Table 3. *Probabilities for emergency department choice for each district during three intervals of the day.*

	D1	D2	D3	D4	D5
ED1 (0-6)	.90	.70	.60	.60	.0
ED2 (0-6)	.10	.30	.28	.26	.10
ED3 (0-6)	0	0	.12	.14	.90
ED1 (6-17)	.80	.70	.80	.60	.0
ED2 (6-17)	.20	.30	.18	.34	.20
ED3 (6-17)	0	0	.02	.06	.80
ED1 (17-24)	.90	.70	.75	.60	.0
ED2 (17-24)	.10	.30	.20	.24	.10
ED3 (17-24)	0	0	.05	.16	.90

The emergency mobile application provides information at discrete levels: the interface broadcast provides comprehensive information about the congestion level and expected service speed of each emergency service in one of three levels: green (G), yellow (Y), or red (R). Figure 3 provides an example of how the status of EDs would appear in the mobile application interface in this case with 3 EDs. The hourly availability condition in the three EDs in this case is encoded as triplets of the letters G, Y, R representing green, yellow and red. Each hour, one of 19 different scenarios can be selected for broadcast (Table 4). It is assumed that the application will display the same scenario to every user, regardless of their district. The effects of these hourly broadcasts, which update every hour, have not yet been tested in practice and are therefore evaluated hypothetically as follows: The GGG broadcast does not alter district routing probabilities. For GGY or YYR broadcasts, traffic directed to the busier emergency services in each district decreases by 20% and is redistributed equally among the two other emergency services. With a GGR broadcast, traffic to the busier service decreases by 30%, redistributing equally to the other services.

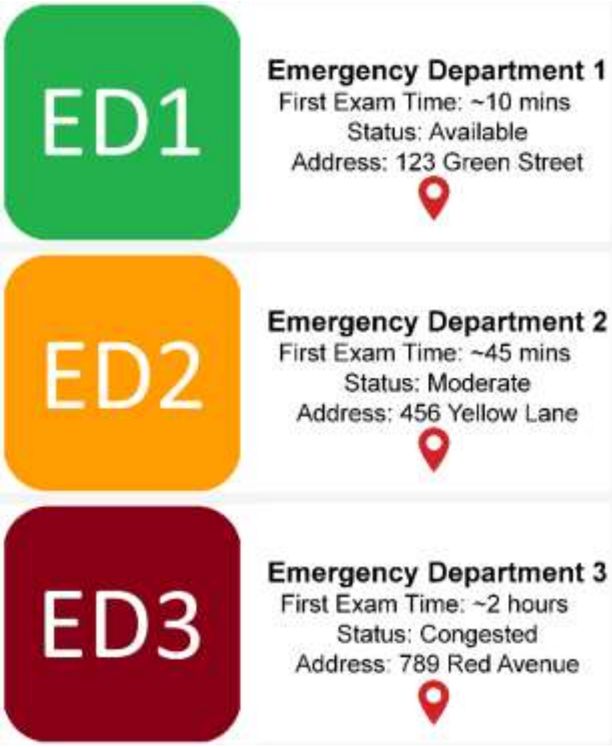

Figure 3. *Illustrative example of the interface of a mobile application displaying the ED status broadcast.*

When a GYY or YRR broadcast is issued, 20% of the traffic from the busier emergency services is redirected to the less congested one. In the case of a GRR broadcast, traffic preferring the busier emergency services decreases by 30% and is directed to the less congested service. For a GYR broadcast, 20% of the traffic from the emergency service marked as Y is redirected to the G service, and 10% of the traffic from the R service is redirected to the Y service. The broadcast colors—green, red, and yellow—do not exactly reflect the current state of the emergency services but rather represent routing decisions derived from optimization for faster service. Emergency requests do not show significant variation or seasonal effects throughout the year, so it would be feasible to set the broadcast for each hour, accounting for the hourly, weekday, and weekend patterns across the 168 hours of a week, and maintain this schedule for several weeks after finding the optimal solution. Each hour having one of the 19 broadcast options as a decision variable creates a vast solution space of size 19^{168}, approximately 6.77×10^{214}. However, with the available simulation environment and optimization tool, it is feasible to optimize within a smaller solution space of size 19^{24}, considering different broadcast policies for all hours of the day. This daily policy is uniformly applied across all days of the week, sacrificing flexibility for improvements regarding traffic durations on different days of the week in exchange for reduced solution time.At each iteration of the heuristic search algorithm, a

Table 4. *Possible states for broadcast by the mobile application. G-green, Y-yellow, R-red.*

Broadcast	# Combinations	Definition
GGG* *YYY RRR*	1	All emergency departments are in a similar crowding state.
GGY *YYR*	3 (GGY, GYG, YGG)	Two emergency departments are slightly more crowded compared to the other.
GYY *YRR*	3 (GYY, YGY, YYG)	Two emergency departments are slightly more crowded compared to the other.
GGR	3 (GGR, GRG, RGG)	One emergency department is significantly more crowded compared to the others.
GRR	3 (GRR, RGR, RRG)	Two emergency departments are significantly more crowded compared to the other.
GYR	6 (GYR, GRY, YGR, YRG, RGY, RYG)	One emergency department has availability, one is slightly crowded while the other is significantly crowded.

* While there is only one combination for YYY -all equal-, there are 6 combinations for GYR broadcast for pointing out availability in different EDs. Broadcast patterns in italic have the same relative effect with the one in bold in the same broadcast group, but the one in bold is preferred in the mobile application interface, signaling that at least one ED is “green”, i.e., welcoming admission in each hour.

broadcast policy is set, and the average patient system time (including travel and care) is measured using a simulation run on the model. A broadcast policy consists of a choice for each hour of the day (broadcast choices in Table 4 form a policy as exampled in Table 5), thus creating a solution space of size 19^{24}.

Defining $\Pi = \{GGG, GGY, GGR, \dots\}$ with $|\Pi| = 19$ as the set of broadcast options available each hour, $x \in \Pi^{24}$ as a fixed policy covering the 24 hours of the day, and $\hat{f}$ as the approximate evaluation of the mean system time for patients under policy x, the simulation-optimization problem can be posed as follows:

$$\min_{x\in\Pi} \hat{f}(x). \qquad (1)$$

The optimization is performed using the heuristic simulation-optimization tool OptQuest of the discrete event simulation software ARENA 14.0. OptQuest, a powerful optimization tool developed by OptTek Systems, Inc., utilizes metaheuristic algorithms to efficiently explore a wide range of potential solutions, identifying the optimal configurations for complex systems [45]. Although the heuristic optimization method and solution ranking and selection method are fixed, parameters can be adjusted for faster or more statistically reliable results [46]. The minimum number of scenarios that satisfy the ranking and selection criteria was set at 3, the simulation warm-up time at 168 hours, and the simulation run time at 840 hours. Currently, selecting the broadcast policy for each hour of the day takes 15 hours on an AMD Threadripper 4.0 GHz processor. Given the limited variation in demand data over the months, a solution can be obtained within a reasonable time using a smaller solution space for 24 daily hours instead of a more flexible 168 weekly hours.

4. Results and Discussion

The simulation-optimization results (Table 5) demonstrate a 10% reduction in the mean total system time, including travel (Table 6). This improvement is primarily attributed to an 11% decrease in the mean service time within the ED. Notably, the optimal solution does not depend on a strategy that reduces travel durations. The optimal broadcast policy significantly reduces waiting times across laboratory, radiology, and consultation stages, as well as for laboratory test results (Table 6). The simulation-optimization solution achieves an approximate 9% reduction in mean laboratory time, with more substantial reductions of 41% and 26% observed in radiology and consultation system times, respectively. Comparing service times between the current setup and the proposed (approximate) optimal solution reveals that redirection to Emergency 2 occurs during two extended periods: from 1 AM to 10 AM and from 2 PM until the end of the day. This strategy effectively balances the lower crowding levels at Emergency 2, consequently reducing the average system time (Table 5).

Table 5. *The best solution obtained by the simulation-optimization run.*

Hour	Broadcast	Hour	Broadcast	Hour	Broadcast
0-1:00	GRG	8-9:00	YGG	16-17:00	GGR
1-2:00	YGR	9-10:00	YGG	17-18:00	GGY
2-3:00	YGR	10-11:00	GYG	18-19:00	GGY
3-4:00	RRG	11-12:00	GYG	19-20:00	GGY
4-5:00	GRY	12-13:00	GYG	20-21:00	GGR
5-6:00	RGR	13-14:00	YYG	21-22:00	GGR
6-7:00	RGR	14-15:00	YGY	22-23:00	YGY
7-8:00	RGR	15-16:00	GGR	23-0:00	YGY

Furthermore, this approach yields an unintended benefit not originally included in the objective function of this study: the utilization rates of physicians are more evenly distributed. The simulation-optimization solution shows that physicians at ED 1, who previously had almost no idle time, experience about a 5% reduction in busy levels. Similarly, physicians at ED 3 see a slight decrease in workload, while ED 2 absorbs additional duties, enhancing the overall balance of physician loads (Table 7).

Table 6. *Average time of patients in travel, laboratory, radiology, consultation stages, the total time in the emergency department and total times in the system including travel, in hours.*

Hours Spent in System/Stages	Travel	Laboratory	Radiology	Consultation	ED Total	System Total
No Broadcast	0.28	4.99	1.07	2.84	6.3	6.59
Simulation-Optimization	0.28	4.56	0.63	2.11	5.6	5.91

The redirection to ED 2 has notably decreased congestion, as reflected in reduced waiting times for initial examinations, result assessment/consultation calls, and consultation queues at ED 1 and ED 3 (Table 8). Given its higher capacity (Table 2), ED 1 received more focus for improvements. Although the instantaneous occupancy rates of doctors at ED 2 have equilibrated with those at the other two facilities (Table 7), the already brief waiting times at ED 2 have slightly increased. Despite this relative increase, the overall enhancements across the system have led to significant reductions in average system times, encompassing both emergency service and traffic times for all emergency departments.

Table 7. *Utilization rates of emergency department physicians.*

	Utilization Rates of Physicians		
	ED1	ED2	ED3
No Broadcast	0.986	0.816	0.958
Simulation-Optimization	0.939	0.920	0.947

Statistical analysis using t-tests on independent measurements (with different random variables generated for each run) confirmed that the total system time, combining traffic and emergency service durations, under the optimal solution ($M = 5.92$, $SD = 0.17$) is significantly shorter than the current setup ($M = 6.59$, $SD = 0.55$), $t(58) = 6.33$, $p < .05$, two-tailed, 95% CI *[0.46, 0.88]*. This improvement holds true solely for emergency service times as well: independent t-tests show that emergency times under the optimal solution (*M = 5.64, SD = 0.17*) are significantly shorter than those in the current situation (*M = 6.31, SD = 0.55*), *t(58) = 6.37, p < .05*, two-tailed, 95% CI *[0.46, 0.88]*.

Table 8. *Waiting times in queues for accessing an emergency physician for tasks*

	Initial Examination		
	ED1	**ED2**	**ED3**
No Broadcast (min)	71.64	12.46	78.64
Simulation-Optimization (min)	25.89	26.84	53.22
	Assessment		
	ED1	**ED2**	**ED3**
No Broadcast (min)	78.18	10.09	81.05
Simulation-Optimization (min)	30.94	24.83	55.73
	Consultation		
	ED1	**ED2**	**ED3**
No Broadcast (min)	77.98	9.45	72.76
Simulation-Optimization (min)	30.91	23.93	51.91

5. Conclusion

This study presents a novel approach for directing emergency department demand by minimizing the total system time experienced by patients. By leveraging a mobile application and considering patient location and ED availability, a simulation-optimization model, which guides patients to the most appropriate location, is proposed, thereby reducing travel time and optimizing health care service delivery.The novelty of this study lies in devising a mobile application that provides real-time information on the availability and crowdedness levels of various emergency departments in each city. This allows patients to make informed decisions about which ED to visit, improving patient distribution across the ED network. The findings highlight the substantial benefits of systematically redirecting patient demand to balance crowding levels across multiple emergency departments within a city or region. The results additionally demonstrate significant improvements in overall system performance, including reduced travel and service times, more balanced workloads among health care providers, and enhanced patient satisfaction.

The implementation of this model can lead to better resource utilization and a more resilient and responsive emergency care network. These managerial insights underscore the importance of integrating real-time data and advanced optimization techniques to achieve efficient and effective emergency department operations. The promising results suggest that adopting such innovative solutions by health care decision and policy makers can significantly enhance the quality and efficiency of emergency medical services, offering a scalable model for broader application in various health care systems.

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