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Research Article

Continuous and Discontinuous Contact Problems of a Functionally Graded Layer [#]

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Abstract: In this study, the continuous and discontinuous contact problem of a functionally graded (FG) layer resting on a rigid foundation is considered. The top of the FG layer is subjected to normal tractions over a finite segment. The graded layer is modeled as a non-homogenous medium with a constant Poisson' ratio and exponentially varying shear modules and density. For continuous contact, the problem is solved analytically using plane elasticity and integral transform techniques. The critical load that causes first separation for various material properties is investigated. The problem is reduced to a singular integral equation using plane elasticity and integral transform techniques in case of discontinuous contact. Obtained singular integral equation is solved numerically using Gauss-Jacobi integral formulation and an iterative scheme is employed to obtain the correct separation distance. The separation distance between the FG layer and the foundation is analyzed. The results are shown in tables and figures. It is seen that decreasing stiffness and density at the top of the layer results an increment in critical load and the lowest pressure occurs on the symmetry axis in case of continuous contact. In addition, the separation distance increases with decreasing stiffness and density at the top of the layer in case of discontinuous contact.

1. Introduction

Problems in which body forces neglected the layer bends and the contact area between the layer and the subspace will diminish to a finite size independent of the magnitude of the applied load. However, particularly in large scale structures and layered media, in practice clearly this may not be the case, that is, in such problems the contact area is expected to depend on the applied load and furthermore away from the loading region the layer will remain in contact with the subspace because of gravity [1].

An examination of the related literature shows that studies involving layers or coatings with body forces have consisted of homogeneous materials [1-3] whereas studies involving FG layers or coating [4-6] have neglected body forces. Therefore, this study aims to solve the continuous and discontinuous contact problem of an FG layer resting on a rigid foundation by taking into account the body force of the FG layer. Further, the calculations are made under the assumption that the FG layer is isotropic and the shear modulus and mass density exponentially vary along the direction of the layer's thickness.

2. Formulation of the Problem

As shown in Fig. 1, consider the symmetric plane strain problem consists of an infinitely long functionally graded (FG) layer of thickness h resting on a rigid foundation. Poisson's ratio is taken as constant, the shear modulus and the density depend on the y-coordinate only as follows:

$$\mu(y) = \mu_0 \exp(\beta y),\ 0 \le y \le h \qquad (1.a)$$

$$\rho(y) = \rho_0 \exp(\alpha y),\ 0 \le y \le h \qquad (1.b)$$

where μ_0 and ρ_0 are the shear modules and the density of the graded layer at $y = 0$, β and α are the non-homogeneity parameter controlling the variation of the shear modules and the density in the graded layer, respectively.

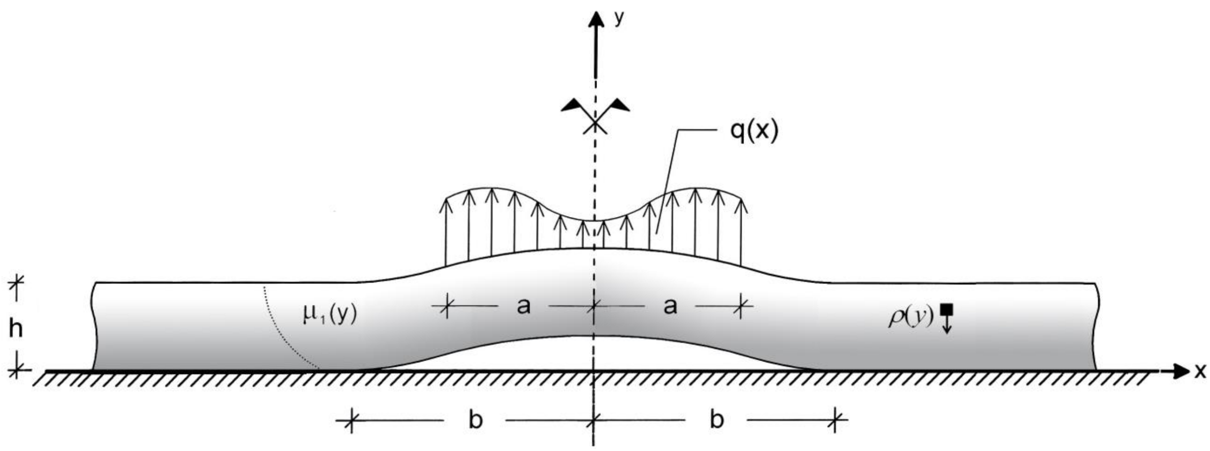

Figure 1. *Geometry and loading of the contact problem*

The top of the layer is subjected to a distributed load $q(x)$ over the segment $|x| \leq a$. It is assumed that the contact surfaces are frictionless and is to be the plane of symmetry with respect to external loads as well as geometry, for simplicity. Clearly, it is sufficient to consider one half (i.e., $x \geq 0$) of the medium only.

If the load is sufficiently small, then the contact between the FG layer and the rigid foundation becomes continuous and the boundary conditions can be written in (2). Otherwise a separation occurs between the FG layer and the rigid foundation in the neighbourhood of $x = 0$ symmetry axis at $y = 0$ and assuming that the separation region is described by $-b < x < b$, the boundary conditions can be shown in (3).

$$\sigma_y(x,h) = q(x)H(a-|x|), \quad \tau_{xy}(x,h) = 0, \qquad (2.a,b)$$

$$v(x,0) = 0, \tau_{xy}(x,0) = 0, 0 \leq x < \infty \qquad (2c,d)$$

$$\sigma_y(x,h) = q(x)H(a-|x|), \tau_{xy}(x,h) = 0, \qquad (3.a,b)$$

$$\frac{\partial v(x,0)}{\partial x} = f(x), \tau_{xy}(x,0) = 0, 0 \leq x < \infty, \qquad (3.c,d)$$

$$\sigma_y(x,0) = 0, -b < x < b \qquad (3e)$$

in which, $f(x)$ is the derivative of separation distance between the layer and the rigid foundation with respect to x and v is the vertical displacement.

For continuous contact, the problem is solved analytically using plane elasticity and integral transform techniques and obtained following equations.

$$\frac{1}{\lambda_{cr}} = \frac{\alpha h}{\exp(\alpha h)-1} \frac{2\exp(-\beta h)}{\pi(a/h)} * \\ * \int_0^\infty \frac{1}{\xi \Delta F} \sum_1^4 \left[F_{j1} S_1 \exp(n_j y)\right] \sin \xi a \cos \xi x d\xi \qquad (4)$$

where, λ_{cr} is the critical load that causes to first separation.

The problem is reduced to a singular integral equation using plane elasticity and integral transform techniques in case of discontinuous contact. Obtained singular integral equation is solved numerically using Gauss-Jacobi integral formulation.

$$\frac{1}{\pi}\int_{-1}^{1} \phi(s)\left[\frac{1}{t-x} + \frac{b}{h}k_1\right] ds + \\ + \frac{1}{a/h \exp(\beta h)} \frac{\lambda}{\pi} k_2 - \frac{\exp(\alpha h)-1}{\alpha h} = 0 \qquad (5)$$

where, k_1 and k_2 are known functions and ϕ is a dimensionless quantity represents the derivatives of separation distance and are obtained using an iterative scheme.

3. Results

The height of the graded layer is taken as 1 whereas the Poison's ratio of the graded layer is taken as 0,25. Note that all quantities are normalized.

Table 1 shows the comparison of λ_{cr} in case of continuous contact and b/h in case of discontinuous contact for homogenous layer, i.e. $\beta = 0.001$ and $\alpha = 0.001$, between the values reported in [1] and obtained in this study . It can be seen that values of b/h and λ_{cr} in this study are approximately the same value given by Civelek and Erdogan [1].

Table 1.*The comparison of λ_{cr} in case of continuous contact and b/h in case of discontinuous case ($a/h = 0.01$).*

This study	λ_{cr}	b/h		
		$\lambda = 1.2$	$\lambda = 2.0$	$\lambda = 4.0$
Civelek and Erdogan [1]	1.088	0.28	0.96	2.35
This study	1.088625	0.287155	0.970928	2.387936

The effect of λ , β and α on contact pressure are shown in Fig. 2.

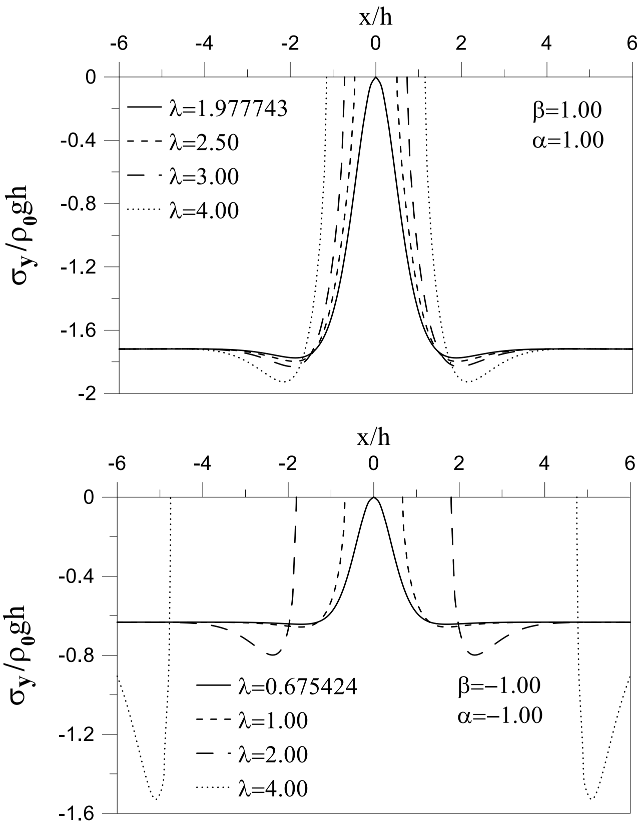

Figure 1. *The effect of λ on contact pressure for various β and α ($a/h = 0.01$)*

It is seen that the separation distance increases with decreasing β and increasing α . In addition, the biggest pressures occur near the end of the separation and decrease for increasing β .

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