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Research Article

Monogenic in N-groups and its Subgroups

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Abstract:

In this article we discuss the concept of monogenic N-groups. First, we introduced the δ-monogenic N-groups and we study its example and properties. Then we define monogenic in proper subgroups of N-groups. We give example of monogenic subgroup of N-groups and structured its properties. Finally, we define the definition of γ-monogenic subgroups of N-groups. We study the example and conditions of γ-monogenic subgroups of N-groups.

1. Introduction

In 1905, Dickson introduced near-field with only one distributive law which is the first near-ring [1]. The term near-ring was first use by Zassenhaus in 1936 [3]. N-group is the analogue of the concept of a module in ring theory. N-group (N-module or near-module) is an additive group which satisfy two conditions with respect to its near-ring. In this article we use the right near-rings and left N-module as an N-groups. The concept of monogenic is similar to the concept of cyclic group. Therefore, every element of monogenic N-groups is the product of elements of near-ring and generator of N-group. Based on the generator of monogenic N-group we first construct the δ-monogenic N-group. Then we define monogenic subgroup and γ-monogenic subgroup of N-group of near-ring. We study their properties through the examples, notes, remarks, propositions and theorems.

2. Preliminaries

Definition 2.1 [2]:

A near-ring is a non-empty set N together with two binary operations " $+$ " and " $\cdot$ " such that

a) $(N, +)$ is a group (not necessarily abelian).
b) $(N, \cdot)$ is a semigroup.
c) $\forall\, n_1, n_2, n_3 \in N$ such that $(n_1 + n_2) \cdot n_3 = n_1 \cdot n_3 + n_2 \cdot n_3$ ("right distributive law").

Class of all near-rings will be denoted by $\mathcal{N}$.

Definition 2.2 [2]:

- $N_0 = \{n \in N \mid n0 = 0\}$ is called the zero-symmetric part of N.
- $N_c = \{n \in N \mid n0 = 0\} = \{\, n \in N \mid \forall\, n' \in N: nn' = n\}$ is called the constant part of N.

N_0 and N_c is itself near-rings.

Definition 2.3 [2]:

$N \in \mathcal{N}$ is called zero symmetric (constant) if $N = N_0$ ($N = N_c$, respectively).

$\mathcal{N}_0$ ($\mathcal{N}_c$) stand for all classes of all zero symmetric (constant) near-rings.

Definition 2.4 [2]:

Let $(\Gamma, +)$ be a group with zero 0 and let $N \in \mathcal{N}$. Let $\mu: N \times \Gamma \longrightarrow \Gamma$. $(\Gamma, +)$ is called an N-group (near-module over N) if $\forall \gamma \in \Gamma\ \ \forall n, n' \in N$:

1. $(n + n')\gamma = n\gamma + n'\gamma$.
2. $(nn')\gamma = n(n'\gamma)$.

We write N^{Γ} for the N-group.

Notation 2.5 [2]:

Let N^{g} be the class of all N-groups.

Definition 2.6 [2]:

A subgroup Δ of N^{Γ} with $N\Delta \subseteq \Delta$ is said to be an N-subgroup of Γ ($\Delta \leq_N \Gamma$).

Definition 2.7 [2]:

Let $N \in \mathcal{N}$. A normal subgroup I of $(N, +)$ is called ideal of N ($I \trianglelefteq N$) if

a) $IN \subseteq I$.
b) For all $n, n' \in N$ and for all $i \in I$ such that $n(n' + i) - nn' \in I$.

Notes 2.8 [2]:

- Normal subgroups R of (N, +) with $RN \subseteq R$ are called right ideals of N ($R \trianglelefteq_r N$).
- Normal subgroups L of (N, +) with $n(n' + l) - nn' \in L$ for all $n, n' \in N$ & $l \in L$ are said to be left ideals of N ($L \trianglelefteq_l N$).

Definition 2.9 [2]:

Let $N \in \mathcal{N}$ and $N^{\Gamma} \in N^{g}$. A normal subgroup Δ of $(\Gamma, +)$ is called ideal of N^{Γ} ($\Delta \trianglelefteq_N \Gamma$) if $\forall \gamma \in \Gamma \; \forall \delta \in \Delta \; \forall n \in N \; such\ that\ n(\gamma + \delta) - n\gamma \in \Delta$.

Remark 2.10 [2]:

- $\{0\}$ and N are ideals of N.
- $\{0\}$ and Γ are ideals of N^{Γ}.

These ideals are called the trivial ideals.

Definition 2.11 [2]:

- N (N^{Γ}) is simple if N (N^{Γ}) has no non-trivial ideals.
- N^{Γ} is called N-simple if N^{Γ} has no N-subgroup except Ω and Γ.

Definition 2.12 [2]:

N^{Γ} is monogenic if there exist $\gamma \in \Gamma$ such that $N\gamma = \Gamma$.

i.e. we say that N^{Γ} is monogenic by γ and γ is called a generator of N^{Γ}.

Example 2.13 [2]:

Let near-ring N and it's N- group N^{Γ} be Klein's four group {0, a, b, c}. It's addition and multiplication are defined by following tables

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	b	0
c	a	a	c	a

{scheme 21, pg.no: 21 in [2]}

N^{Γ} is monogenic by $\gamma = b$.

Definition 2.14 [2]:

N^{Γ} is strongly monogenic if N^{Γ} is monogenic and $\forall \gamma \in \Gamma$ such that $N\gamma = \{0\}$ or $N\gamma = \Gamma$.

Definition 2.15 [2]:

N^{Γ} with $N \in \mathcal{N}_0$ is called completely reducible if N^{Γ} is the direct sum of simple ideals.

Note 2.16 [2]:

$N = N_0 \Leftrightarrow (\Delta \trianglelefteq_N \Gamma \Rightarrow \Delta \leq_N \Gamma)$ for all $\Gamma \in N^{g}$.

Remark 2.17 [2]:

Let N^{Γ} be monogenic (by γ). Then $L \trianglelefteq_l N \Longrightarrow L\gamma \trianglelefteq_N \Gamma$.

Note 2.18 [2]:

$\forall \gamma \in \Gamma : (0:\gamma) \trianglelefteq_l N$.

3. Main Results

3.0. Example for Monogenic N-group

Remark 3.0.1:

Monogenic N-group has more than one monogenic generator.

Example 3.0.2:

Let $N = Z_6$ be a near-ring and $N^{\Gamma} = Z_6$ be a N-group under addition modulo 6 and multiplication is defined by

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

{scheme 27, pg. no: 409 in [2]}

$N^{\Gamma} = Z_6$ is monogenic by $\gamma = 1\ and\ 5$.

$\therefore$ $N^{\Gamma} = Z_6$ has more than one monogenic generator.

Remark 3.0.3:

By note 2.16,

If $N \in \mathcal{N}_0$ then every ideal of N^{Γ} are N-subgroups of N^{Γ}.

Theorem 3.0.4:

If N^{Γ} is monogenic by γ_0 then $(0:\gamma)\gamma_0 \trianglelefteq_N N^{\Gamma}$, for all $\gamma \in \Gamma$.

Proof:

Let Γ be a monogenic N-group by $\gamma_0 \in \Gamma$.

By note 2.18,

$\Longrightarrow (0:\gamma) \trianglelefteq_l N$, for all $\gamma \in \Gamma$.

By remark 2.17,

$\Longrightarrow (0:\gamma)\gamma_0 \trianglelefteq_N N^{\Gamma}$, for all $\gamma \in \Gamma$.

Note 3.0.5:

Every subgroup (ideal, N-subgroup) of monogenic N-group N^{Γ} need not be monogenic.

3.1. δ-Monogenic:

Definition 3.1.1:

An N-group Γ is called δ-monogenic if there exists $\delta \in \Delta$ where Δ is a proper subgroup of N^{Γ}such that $N\delta = \Gamma$.

δ is a called δ-generator or δ-monogenic generator of N^{Γ}in its proper subgroup Δ.

Example 3.1.2:

Let $N = Z_8$ be a near-ring under addition modulo 8 and multiplication is defined as

.	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	0	2	4	6
3	0	3	6	1	4	7	2	5
4	0	4	0	4	0	4	0	4
5	0	5	2	7	4	1	6	3
6	0	6	4	2	0	6	4	2
7	0	7	6	5	4	3	2	1

{Scheme 46, pg.no: 413 in [2]}

Then $\Gamma = Z_8$ is a δ-monogenic N-group of $N = Z_8$. In which $\delta = 1,7 \in \Delta = \{0, 1, 7\}$ are δ-generators of N^{Γ}.

Remarks 3.1.3:

- By definition 3.1.1, $N\delta = \Gamma$, δ is monogenic generator of N-group N^{Γ} in its subgroup Δ.
- There is more than one δ-monogenic generator of N-group N^{Γ} in its subgroup Δ. The above example also confirmed it.
- Not every proper subgroup of N^{Γ} has δ-generator of N^{Γ}. In above example the proper subgroup Δ = {0, 2, 6} of N-group $\Gamma = Z_8$ has no δ-generator of N-group $\Gamma = Z_8$.

Note 3.1.4:

If N^{Γ} is a δ-monogenic by $\delta \in \Delta$, then any other proper subgroup Δ' of N^{Γ} which has no δ-monogenic generator is not an ideal.

Example 3.1.5:

For the near-ring $N = Z_6$ under addition modulo 6 and multiplication is defined as

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	3	5	5	3	1	1
2	0	4	4	0	2	2
3	3	3	3	3	3	3
4	0	2	2	0	4	4
5	3	1	1	3	5	5

{scheme 24, pg.no: 409 in [2]}

Let $\Gamma = Z_6$ be the N-group of near-ring $N = Z_6$. Let $\Delta = \{0, 2, 4\}$ and $\Delta' = \{0, 3\}$ be the proper subgroups of N-group $\Gamma = Z_6$. Here $\Gamma = Z_6$ is δ-monogenic by $\delta = 4 \in \Delta$. Also, none of elements of δ-generator of $\Gamma = Z_6$. Then Δ' is not an ideal.

Proposition 3.1.6:

If N^{Γ} is δ-monogenic then N^{Γ} is monogenic.

Proof:

Let $N \in \mathcal{N}$

Let Γ be an N-group of the near-ring N.

Given Γ is δ-monogenic.

By definition of δ-monogenic,

there exist $\delta \in \Delta$ where Δ is a proper subgroup of Γ such that $N\delta = \Gamma$

Since $\Delta \subset \Gamma$,

there exist $\delta \in \Delta \subset \Gamma$ such that $N\delta = \Gamma$

$\Rightarrow$ there exist $\delta \in \Gamma$ such that $N\delta = \Gamma$.

Therefore, N^{Γ} is monogenic.

Remark 3.1.7:

Converse is not true

If N^{Γ} is monogenic then N^{Γ}need not be δ-monogenic.

Example 3.1.8:

For the near-ring $N = Z_6$ under addition modulo 6 and multiplication is defined as

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

{scheme 27, pg.no: 409 in [2]}

Let $\Gamma = Z_6$ be the N-group of near-ring $N = Z_6$. Let $\Delta = \{0, 2, 4\}$ and $\Delta' = \{0, 3\}$ be the proper subgroups of N-group $\Gamma = Z_6$. In which Δ and Δ' has no δ-generator of N-group $\Gamma = Z_6$. Then $\Gamma =$

Z_6 is monogenic N-group, but $\Gamma = Z_6$ is not δ-monogenic.

Note 3.1.9:

If N^{Γ} is δ-monogenic by δ, then N^{Γ} is also monogenic by δ.

Proposition 3.1.10:

If N^{Γ} is a δ-monogenic by δ then every element of N^{Γ} is of the form nδ, for any $n \in N$.

Proof:

Let N be a near-ring.

Let Γ be an N-group of N.

Also, Γ is δ-monogenic by δ.

There exist $\delta \in \Delta$ where Δ is proper subgroup of Γ such that $N\delta = \Gamma$.

For all $\gamma \in \Gamma$,

$\gamma = n\delta$, for any $n \in N$.

Hence, every element of Γ is of the form $n\delta$ for any $n \in N$.

Proposition 3.1.11:

Let N^{Γ} be δ-monogenic by δ If Δ is a proper simple ideal of N^{Γ} and $\delta \in \Delta$, then Δ is the only simple ideal of N^{Γ}.

Proof:

Given Δ is a simple ideal of Γ.

Let Γ be δ-monogenic by δ.

Let $\delta \in \Delta \subset \Gamma$ and Δ be a simple ideal of Γ.

Therefore, there exist $\delta \in \Delta$ such that $N\delta = \Gamma$.

To prove, Δ is the only simple ideal of Γ.

Suppose Δ' is another disjoint simple ideal of Γ.

By definition of ideal,

$n(\gamma + \delta') - n\gamma \in \Delta'$, for all $n \in N, \gamma \in \Gamma, \delta' \in \Delta'$

By proposition 3.1.10,

Since $\delta' \in \Delta' \subseteq \Gamma$, $\delta' \in \Gamma$.

$\Longrightarrow \delta' = n_1\delta$, for any $n_1 \in N$.

Also $\gamma \in \Gamma \Longrightarrow \gamma = n_2\delta$, for any $n_2 \in N$.

Now

$n(\gamma + \delta') - n\gamma \in \Delta$

$\Rightarrow n(n_1\delta + n_2\delta) - n(n_2\delta) \in \Delta'$

$\Rightarrow n\big((n_1 + n_2)\delta\big) - (nn_2)\delta \in \Delta'$

$\Rightarrow n(n_3\delta) - (nn_2)\delta \in \Delta'$

$\Rightarrow (nn_3)\delta - (nn_2)\delta \in \Delta'$

$\Rightarrow n_4\delta - n_5\delta \in \Delta'$

$\Rightarrow n_4\delta + n_6\delta \in \Delta'$

$\Rightarrow (n_4 + n_6)\delta \in \Delta'$

$\Rightarrow n_7\delta \in \Delta'$

But, $n_7\delta \in N\delta$, for any $n_7 \in N$

Since N^{Γ} is δ-monogenic, $n_7\delta \in N\delta = \Gamma$

$\Rightarrow n_7\delta \in \Gamma$.

Therefore, $\Gamma \subseteq \Delta'$.

Clearly $\Gamma \not\subset \Delta'$.

The only possibility is $\Gamma = \Delta'$

i.e. Δ' is a trivial ideal.

which is a contradiction.

our assumption is wrong.

Hence Δ is the only simple ideal of Γ.

3.2 Monogenic subgroup:

Definition 3.2.1:

A proper subgroup Δ of N^{Γ} is called monogenic if there exists $\delta \in \Delta$ such that $N\delta = \Delta$.

Therefore δ is a generator of Δ or Δ-generator of Γ.

Example 3.2.2:

Let $N = Z_6$ be a near-ring and $N^{\Gamma} = Z_6$ be a N-group under addition modulo 6 and multiplication is defined by

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	4	2	0	4	2
2	0	2	4	0	2	4
3	0	0	0	0	0	0
4	0	4	2	0	4	2
5	0	2	4	0	2	4

{scheme 36, pg. no: 409 [2]}

$\Delta = \{0, 2, 4\}$ is the monogenic subgroup of $\Gamma = Z_6$ where $\delta = 2, 4$ are generators of Δ or Δ-generator of Γ.

Note 3.2.3:

From the above example 3.2.2, there are more than one generator of Δ or Δ-generator of the N-group Γ.

Lemma 3.2.4:

Let $\Delta \subset N^{\Gamma}$. If Δ is monogenic by δ then every element of Δ is of the form $n\delta$, for any $n \in N$.

Proof:

Let Γ be an N-group of the near-ring N.

Given Δ is a proper subgroup of N-group Γ.

Let Δ be a monogenic subgroup by δ.

$\Rightarrow N\delta = \Delta$.

$\Rightarrow$ for all $\delta_1 \in \Delta$, $\delta_1 = n\delta$ for any $n \in N$.

Hence every element of Δ is of the form $n\delta$, for any $n \in N$.

Theorem 3.2.5:

If Δ is a monogenic subgroup of N^{Γ} then Δ is an N-subgroup.

Proof:

Let Γ be an N-group of the near-ring N.

Let Δ be a proper subgroup of Γ.

Given that, Δ is a monogenic subgroup of Γ.

There exist (say) $\delta \in \Delta$ such that $N\delta = \Delta$.

To prove Δ is an N-subgroup.

i.e. to prove $N\Delta \subseteq \Delta$.

Let $x \in N\Delta$.

$\Rightarrow x = n\delta_1$, for any $n \in N$ and $\delta_1 \in \Delta$

Since $\delta_1 \in \Delta$, $\delta_1 = n'\delta$ for any $n' \in N$.

Now, $x = nn'\delta$ for any $n, n' \in N$.

$\Rightarrow x = n_1\delta$, for $n_1 \in N$.

Therefore, x is of the form $n\delta$.

$\Rightarrow x \in \Delta$.

Thus $N\Delta \subseteq \Delta$.

Hence Δ is an N-subgroup.

Theorem 3.2.6:

If N^{Γ} is monogenic and it has a monogenic subgroup then monogenic generator of subgroup is of the form $n\gamma$, for any $n \in N$.

Proof:

Let Γ be a monogenic N-group.

There exist $\gamma \in \Gamma$ such that $N\gamma = \Gamma$.

Every element of Γ is of the form $n\gamma$, for any $n \in N$.

Let Δ be a monogenic subgroup of Γ.

There exist $\delta \in \Delta$ such that $N\delta = \Delta$.

Since $\Delta \subset \Gamma$, $\delta \in \Gamma$.

Therefore, δ is of the form $n\gamma$, for any $n \in N$.

Note 3.2.7:

From above example 3.3.2, we get, if $\Delta \subset N^{\Gamma}$ is monogenic then N^{Γ} need not be monogenic.

Theorem 3.2.8:

Let $\Delta \subset N^{\Gamma}$. If Δ is monogenic by $\delta \in \Delta$, then none of the elements of Δ are δ-generator of N^{Γ}.

Proof:

Let Γ be an N-group.

Given $\Delta \subseteq \Gamma$.

Let Δ be monogenic by $\delta \in \Delta$.

$\Rightarrow N\delta = \Delta$. ---------- (1)

To prove, none of the elements of Δ are δ-monogenic generator of N^{Γ}.

Suppose Δ has δ-monogenic generator of Γ.

Let δ' be the δ-monogenic generator of Γ in Δ.

$\Rightarrow N\delta' = \Gamma$

Hence every element of Γ is of form $n\delta'$, for any $n \in N$.

Since $\delta \in \Delta \subseteq \Gamma$, $\delta = n\delta'$ for any $n \in N$.

$(1) \Rightarrow Nn\delta' = \Delta$.

$\Rightarrow N\delta' = \Delta$.

$\Rightarrow \Delta = \Gamma$.

Which is a contradiction to proper subgroup Δ.

Our assumption is wrong.

Therefore, none of the elements of Δ are monogenic generator of N^{Γ}.

Theorem 3.2.9:

Let δ be a Δ-generator of N^{Γ}. If I is the left ideal of the near-ring N then $I\delta$ is an ideal of subgroup Δ of N^{Γ}.

Proof:

Let I be the left ideal of the near-ring N.

Let Δ be proper subgroup of N-group Γ.

Given δ is a Δ-generator of Γ.

i.e. δ is generator of the subgroup Δ of Γ.

$\Rightarrow \Delta$ is monogenic by δ.

By lemma 3.2.4, for all $\delta_1 \in \Delta$, $\delta_1 = n'\delta$ for any $n' \in N$.

To prove $I\delta$ is an ideal of Δ.

Now

for all $i\delta \in I\delta$ and $n \in N$ and $\delta_1 \in \Delta$,

$$n(\delta_1 + i\delta) - n\delta_1 = n(n'\delta + i\delta) - n(n'\delta)$$

$$= n((n' + i)\delta) - (nn')\delta$$

$$= (n(n' + i) - (nn'))\delta$$

$$\in I\delta.$$

therefore $n(\delta_1 + i\delta) - n\delta_1 \in I\delta$.

hence $I\delta$ is an ideal of the proper subgroup Δ of N-group Γ.

3.3. γ-Monogenic Subgroup:

Definition 3.3.1:

A proper subgroup Δ of an N-group N^{Γ} is said to be γ-monogenic if there exists $\gamma \in \Gamma$, but $\gamma \notin \Delta$ such that $N\gamma = \Delta$.

γ is called γ-generator of Δ in Γ.

Example 3.3.2:

Let $N = Z_8$ be a near-ring under addition modulo 8 and multiplication is defined as

.	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	2	4	4	0	2	0	4
2	0	4	0	0	0	4	0	0
3	0	6	4	4	0	6	0	4
4	0	0	0	0	0	0	0	0
5	0	2	4	4	0	2	0	4
6	0	4	0	0	0	4	0	0
7	0	6	4	4	0	6	0	4

{Scheme 112, pg.no: 414 in [2]}

$\Delta = \{0, 2, 4, 6\}$ is the γ-monogenic subgroup of $\Gamma = Z_6$. $\gamma = 1, 5$ are γ-generator of Δ or γ-generator of Γ.

Note 3.3.3:

From above example 3.3.2

i. There are more than one γ-generator of Δ in Γ.
ii. If Δ is a γ-monogenic subgroup of N^{Γ}, then Δ need not be a monogenic subgroup of N^{Γ}.
iii. Every $\gamma \notin \Delta$ need not be a γ-generator of Δ in N^{Γ}.

Theorem 3.3.4:

Let $\Delta \subset N^{\Gamma}$. If $\gamma \in \Gamma$ is a γ-generator of Δ then every element of Δ is of the form $n\gamma$, for any $n \in N$.

Proof:

Let Γ be an N-group of near-ring N.

Given $\Delta \subset \Gamma$.

Let γ be a γ-generator of Δ.

$\Rightarrow N\gamma = \Delta$.

$\Rightarrow$ for all $\delta \in \Delta$,

$$\delta = n\gamma, \text{ for any } n \in \mathrm{N}.$$

Hence every element of Δ is of the form $n\gamma$, for any $n \in \mathrm{N}$.

Corollary 3.3.5:

If $\gamma \in \Gamma$ is a γ-generator of Δ then there is an element in Γ which is not in the form of $n\gamma$, for any $n \in \mathrm{N}$.

In above example 3.3.2, $\gamma = 7$ is not in the form of product of $n \in \mathrm{N}$ and γ-generator of Δ.

Theorem 3.3.6:

Let Δ be a γ-monogenic subgroup of N^{Γ}. If N^{Γ} is monogenic by γ, then γ-generator of Δ is of the form $n\gamma$, for any $n \in \mathrm{N}$.

Proof:

Let Γ be an N-group of near-ring N.

Let Γ be monogenic by γ.

$\Rightarrow \mathrm{N}\gamma = \Gamma$.

i.e. every element of Γ is of the form $n\gamma$, for any $n \in \mathrm{N}$.

Let $\Delta \subset \Gamma$.

Given Δ is a γ-monogenic subgroup of Γ.

Therefore, there exist $\gamma' \in \Gamma$ but $\gamma' \notin \Delta$ such that $\mathrm{N}\gamma' = \Delta$.

Since $\gamma' \in \Gamma$, $\gamma' = n\gamma$ for any $\gamma \in \Gamma$.

Hence, γ-generator of Δ is of the form $n\gamma'$, for any $n \in \mathrm{N}$.

Corollary 3.3.7:

Let Δ be a γ-monogenic subgroup of N^{Γ}. If N^{Γ} is monogenic by γ, then every element of Δ is of the form $n\gamma$, for any $n \in \mathrm{N}$.

Note 3.3.8:

A proper subgroup Δ of N^{Γ} is monogenic and γ-monogenic but its generator and γ-generator of Δ are distinct.

Example 3.3.9:

Let $\mathrm{N} = Z_6$ be a near-ring and $\mathrm{N}^{\Gamma} = Z_6$ be a N-group under addition modulo 6 and multiplication is defined by

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	3	0	3	0	3
2	0	0	0	0	0	0
3	0	3	0	3	0	3
4	0	0	0	0	0	0
5	0	3	0	3	0	3

{scheme 55, pg. no: 410 in [2]}

$\Delta = \{0, 3\}$ is the monogenic and γ-monogenic subgroup of $\Gamma = Z_6$. But $\delta = 3$ is generator of Δ and $\gamma = 1, 5$ are γ-generator of Δ in Γ.

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