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**Research Article**



# Elastic Riemannian Framework for Whole-to-Part Shape Registration

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**Abstract:**

In this paper, we focus on one of partial shape analysis tasks, by proposing a solution for the whole-to-part shape registration problem using a Riemannian Framework extended from past works. We have a part of a planar curve considered as open one, and the goal is to extract an open curve segment from another closed curve that gives the best alignment by minimizing the Riemannian shape distance between the given part and all parts of the closed curve with the same length. Our contribution here is: (1) extracting all curves with same length of the target curve from the second by changing the starting point, (2) for each part extracted, find the best registration using an elastic Riemannian metric and calculating the shape distance between them (3) the minimum of these distances define the distance desired and the curve is the part which gives this distance. Finally, to evaluate the performance of this algorithm, the results are shown in application to shapes from MPEG-7 dataset.

## 1. Introduction

Whole-to-part shape matching is a fundamental task in partial shape analysis arise in wide range of application domains, for example, Robotic manipulation where identifying object to manipulate especially when part of this object is hidden, in medical imaging as needed in Surgical planning, this process involves aligning pre-operative and intra-operative data which can be partial or deformed, and when we detect the contours of organs are obscured by other structures,
in computer graphics, which require aligning deformable objects with only a portion of their contours or surfaces matching and in handwriting text recognition, analyse the characters as partial planar curves by map, segment and classify them. The problem addresses the issue of registering or aligning a particular part of a curve with a part from another closed or open curve.

Several papers represent many methods to solve this critical problem with different shape representations and different algorithms for matching. "T.B. Sebastian et al. (2003) present an approach to aligning pairs of curves segments and pairs of closed curves by a metric defined with length and curvature of these curves and the results calculated by dynamic-programming method in [11], [9] calculate the distance between two trajectories P and Q by finding a path that minimizes the average Euclidean distance between all points of P and Q. In [10], T. Richardson et al. (2006) present an algorithm where one-to-one correspondence does not exist across the shapes and this algorithm initialize to clip some portion of open-curves and then apply the Landmark-sliding algorithm to find the correspondence from each starting clip length. The goal of T. Funkhouser et al. (2006) in [3] was to retrieve 3D models from a given database most similar to a 3D model provided as a query by representing every object by a set of local shape features and applying the Priority—driven search algorithm to find the best multi-feature matches without computing complete matches for every object. A new approach proposed in [5] to perform partial shape matching by transform shapes into sequences and utilize an algorithm that determines a subsequence of a target sequence that best match a query where this algorithm map this issue to the cheapest path in a directed acyclic graph DAG and the procedure allows to compute the optimal scale and translation of sequence values, M. Cui et al. (2009) in [2], gives a framework for planar open 2D curves with new signature based on the integral of unsigned curvature and a parameterization that is invariant under similarity transform: translation, rotation and uniform scaling to find what part of the first matches the best with a part or the whole of the second curve, while in [1], the first paper which study the partial similarity in the continuous setting with all points in the curves considered, K. Buchin et al. (2009) study the partial similarity between two polygonal curves of sizes $n$ and $m$ respectively by maximizing the total length of sub-curves that are closed to each other, the closeness is measured by the Fréchet distance where the maximal length is called the partial Fréchet similarity between the two input curves and solve this problem in O(mn(m+n)log(mn)) time. Later, A. Maheshwari et al. (2011) in [6] improved algorithms for partial curve matching, closed Fréchet distance, maximum walk problem and matching a curve in DAG the results based on new data structure called free-space map achieving a running time of O(mn) [8] gives a new technique to the problem of partial shape matching between an open contour and a part of a closed contour using a novel shape descriptor based on measuring the distance of a certain silhouette point from the closest points of the same silhouette along properly defined directions. The shape descriptors are computed along open or closed contours in a spatially non-uniform manner and the resulting collections of of shape descriptors constitute the global shape representation. Whole-to-part and part-to-part matching are discussed in [15], proposing a novel shape descriptor, triangular centroid distances TCDs for shape representation and is invariant to translation, rotation, scaling and considerable shape deformations. The main framework in shape analysis of 2D curves used in this paper is from [14] and sufficient concepts, definitions with details and breadth are in [13], the shapes are continuous boundaries defined by parameterized curves which are elements of the shape space and the essential representation of curves is the Square-Root Velocity Function (SRVF). In this paper, our goal is to best matching an open curve to a part of a closed curve so we will extract the optimal open curve from a larger curve to register it with the target curve using theses concepts in [12]. In [4], the authors represent a good treatment and results for handwritten word segmentation into individual characters and partial matching of individual letters with entire words using elastic Riemannian shape metric. [7] develop an algorithm for shape-informed partial matching and completion with respect to a complete template curve.

In this paper, our goal is to find the best registration between a given open curve and a part of a closed curve. First, measuring the length of the target open curve than extracting the open curves with same length from the closed curve by changing the starting point along it and after that, find the best registration between the target curve and the extracted one and measuring the Riemannian distance between them where the minimum of these distances gives the optimal part which gives the best registration. Finally, applying this approach to 2D planar curves from the MPEG-7 dataset yo show the efficiency of the contribution.

## 2. Preliminaries

Before we construct our procedure, we resume the past Riemannian framework of elastic shape analysis of planar curves using the Square–Root Velocity representation and supposing that the parameter interval is $I = [0; 1]$ following [13] where there is more details. Let $\mathrm{C} = \{\beta : [0; 1] \longrightarrow \mathbb{R}^2 / \beta \text{ absolutely continuous}\}$ be the space of planar parameterized curves, we represent the curve $\beta$ by its Square-Root Velocity function (SRVF) q defined by $q(t) = \dot{\beta}(t)/\sqrt{\|\dot{\beta}(t)\|}$ where $q \in L^2([0; 1]; \mathbb{R}^2)$ and for a differentiable curve $\beta$ its length is $L[\beta] = \int_0^1 \sqrt{\langle \dot{\beta}(t); \dot{\beta}(t) \rangle} dt = \int_0^1 |\dot{\beta}(t)| dt = l,$ we obtain $\|q\|^2 = \int_0^1 |q(t)|^2 dt = \int_0^1 |\dot{\beta}(t)| dt = l.$ Since $t$ varies from 0 to 1, the point $\beta(t)$ traces a path $\beta(0)$ to $\beta(1)$. $\Gamma_{\mathrm{I}} = \{\gamma \in \mathrm{Diff}([0; 1]) / \gamma(0) = 0; \gamma(1) = 1;\ \dot{\gamma}(\mathrm{t}) > 0\}$ note the group of orientation-preserving diffeomorphisms of $[0; 1]$ called the group of re- parameterization functions. The right action of $\Gamma_{\mathrm{I}}$ on $C$ defined by $(\beta, \gamma) \mapsto \beta \circ \gamma$, here the composition $\beta \circ \gamma$ is the re-parameterization of the curve $\beta$ which do not change its shape.

To analyze shapes of curves, its representation must be invariant to scaling, translation, rotation and re-parameterization so we need to study the affectation of these transformations on the curves, this means the action of each group transformation on $C$. First, we will scale all curves to be of a fixed length supposed 1 we get $\|q\|^2 = \int_0^1 |\dot{\beta}(t)| dt = 1,$ then and by using $\dot{\beta}(t)$ in the SRV representation, the translation is removed , for removing rotation and re-parameterization we will use the notion of *pre-shape* space.

The pre-shape space of fixed-length absolutely continuous curves under arbitrary parameterizations using the SRV representation defined on $[0;1]$ is $S = \left\{q \in L^2([0;1];\mathbb{R}^2) / \int_0^1 |q(t)|^2 dt = 1\right\}$, it is a sphere in $L^2([0;1];\mathbb{R}^2)$ and constitute an orthogonal section of scaling group $\mathbb{R}^\times$ on $L^2([0;1];\mathbb{R}^2)$. Under the $scaled - L^2$ metric the action of $\mathbb{R}^\times$ establish an isometric map between $S$ and $L^2([0;1];\mathbb{R}^2)/\mathbb{R}^\times$ thus we will use $S$ for analyzing the elements of $L^2([0;1];\mathbb{R}^2)/\mathbb{R}^\times$. Now we construct the Riemannian structure on $S$ establishing the tangent space and define the inner product on it.

Define the mapping $\Phi: L^2([0;1];\mathbb{R}^2) \longrightarrow \mathbb{R}$ by $\Phi(q) =$ then $S = \Phi^{-1}(1)$ using the linear transformation $d(\Phi)_2: L^2([0;1];\mathbb{R}^2) \longrightarrow \mathbb{R}$ defined by:$d(\Phi)_q = \frac{d}{d\varepsilon}\Big|_{\varepsilon=0} \Phi(q + \varepsilon w) =$ $2\int_0^1 \langle w(t), q(t)\rangle dt = 2\langle q, w\rangle$ to prove that $S$ is a submanifold of $L^2([0;1];\mathbb{R}^2)$, the tangent space $T_q(S)$ is given by:

$T_q(S) = \ker\left(d(\Phi)_q\right) = \{w \in L^2([0;1];\mathbb{R}^2)/ \langle q, w\rangle = 0\}$, the standard $L^2$ inner product between $u, v \in T_q\left(L^2([0;1];\mathbb{R}^2)\right)$, is defined by $\langle u, v\rangle = \int_0^1 \langle u(t), v(t)\rangle dt$ the normal space of $S$ at $q$ is $N_q(S) = span(q)$. Let $\alpha: [0;1] \longrightarrow \mathbb{R}$ be a parameterized path on $S$ that is differentiable everywhere on $[0;1]$ its length is $L[\alpha] = \int_0^1 \sqrt{\langle \frac{d\alpha}{d\tau}, \frac{d\alpha}{d\tau}\rangle}\, d\tau$, $S$ is a Riemannian manifold with the Riemannian metric inherited from the larger Hilbert space $L^2([0;1];\mathbb{R}^2)$, the minimal geodesics are given by the shorter arcs on great circles, for two parameterized curves $\beta_1$ and $\beta_2$, the geodesic path between their representations $q_1$ and $q_2$ is:

$\alpha(\tau) = \frac{1}{\sin(\vartheta)}\left[\sin\left(\vartheta(1-\tau)\right)q_1 + \sin(\tau\vartheta)q_2\right]$ where $\vartheta = \cos^{-1}(\langle q_1, q_2\rangle)$ starts at $q_1$ at $\tau = 0$ and achieve $q_2$ at $\tau = 1$ and the geodesic distance is : $d_S(q_1, q_2) = \vartheta = \cos^{-1}(\langle q_1, q_2\rangle)$ the exponential map, $exp: T_q(S) \to S$ is given by $exp_q(w) = \cos(\|w\|)q + \sin(\|w\|)\frac{w}{\|w\|}, w \in T_q(S)$ is a tangent direction, and the for any $q_2 \in S$ the inverse of the exponential map at $q_1 \in S$ is $\exp_{q_1}^{-1}: S \longrightarrow T_{q_1}(S)$ defined by: $v = \frac{\vartheta}{\sin(\vartheta)}(q_2 - \cos(\vartheta)q_1)$, $\vartheta = \cos^{-1}(\langle q_1, q_2\rangle)$.

The right group action of $\Gamma_I$ on $S$ defined by $(q, \gamma) \longmapsto (q \circ \gamma) \longmapsto (q \circ \gamma)\sqrt{\dot{\gamma}}$ and $(q \circ \gamma)\sqrt{\dot{\gamma}}$ is the representation of the re-parameterized curve $\beta \circ \gamma$. The action of the rotation group $SO(2)$ on $C$ is $(O, \beta) = O\beta$ and represented by the action of $SO(2)$ on $S$ is $(O, q) \longmapsto \{t \longmapsto Oq(t)\}$. Since the actions of $SO(2)$ and $\Gamma_I$ commute we have:

we have for every $q \in S$: $[q] = \left\{Oq\left(\gamma(t)\right)\sqrt{\dot{\gamma}(t)}/ (\gamma, O) \in \Gamma_I \times \mathrm{SO}(2)\right\}$. The main importance of the SRVF is that $\Gamma_I \times \mathrm{SO}(2)$ acts by isometries on $S$, that is mean:$\|q_1 - q_2\| = \left\|\sqrt{\dot{\gamma}(t)}O(q_1 \circ \gamma) - \sqrt{\dot{\gamma}(t)}O(q_2 \circ \gamma)\right\|$ with respect to the chosen metric, and because of this property, the $L^2$ norm between the SRVFs is called ***elastic Riemannian metric***. The shape space using the SRVF representation is defined by: $\chi = S/\left(\Gamma_I \times \mathrm{SO}(2)\right)$ and it is a metric space with the distance inherited from $S$ which defined by: $d_\chi([q_1],[q_2]) =$ $\inf_{\tilde{q}_1 \in [q_1], \tilde{q}_2 \in [q_2]} d_S(\tilde{q}_1, \tilde{q}_2) =$ $\inf_{(\gamma, O) \in \Gamma_I \times \mathrm{SO}(2)} d_S\left(q_1, O(q_2 \circ \gamma)\sqrt{\dot{\gamma}(t)}\right)$

To calculate the shape distance, we solve, over the space $\Gamma_I \times \mathrm{SO}(2)$, the optimization problem

$$(\gamma^*, O^*) = argmin_{(\gamma, O) \in \Gamma_I \times \mathrm{SO}(2)} \cos^{-1}\langle q_1, O(q_2 \circ \gamma)\sqrt{\dot{\gamma}(t)}\rangle = argmin_{(\gamma, O) \in \Gamma_I \times \mathrm{SO}(2)} \left\|q_1 - O(q_2 \circ \gamma)\sqrt{\dot{\gamma}(t)}\right\|^2 \quad (1)$$

**3. Partial Shape Analysis of 2D Planar Curve**

Let $\beta_1$, $\beta_2$ be two unit-length parameterized planar closed curves defined on $[0;1]$, $\beta_O$ is a part of $\beta_2$ defined on $[0;c]$ where $0 < c < 1$. We have $L[\beta_O] = \int_0^c \left|\dot{\beta}_O(t)\right| dt = |\beta_O(c)|$

and $0 < |\beta_O(c)| < 1$.

To obtain the best alignment of $\beta_O$ from $\beta_2$ we propose the following algorithm:

1. Normalize $[0;c]$ to $[0;1]$, and rescale $\beta_O$ to be with unit-length.
2. Construct a sequence of $N+1$ parameters from $[0;1]$ such that: $t_0 = 0$, $t_1 = \frac{1}{N}, \dots, t_N = 1$ and sub-domains with length equal to $c$ defined by $[t_i; t_i + c] \subset [0;1], 0 < t_i + c \leq 1$ for $i = \overline{0, N}$.
3. For each $i = \overline{0, N}$, extracting an open curve $\beta_i$ from $\beta_2$ such that $\beta_i = \beta_2|_{[t_i; t_i + c]}$ .
4. For each $i = \overline{0, N}$ normalize each sub-domain to $[0;1]$ and rescale each curve to be with unit length.

5. The SRVF of $\beta_O$ and $\beta_i$ for each $i = \overline{0,N}$ noted $q_O$ and $q_i$ respectively are elements in $S$ since $\beta_O$ and $\beta_i$ are absolutely continuous planar parameterized open curve.
6. Applying the Riemannian Framework proposed in [14] for each $i = \overline{0,N}$, to find optimal rotation $O_i^*$ and optimal re-parameterization $\gamma_i^*$ by solving (1) for $q_O$ and $q_i$ so we find $q_i^* = \sqrt{\dot{\gamma}_i^*} O_i^* (q_i \circ \gamma_i^*)$, and the shape distance is $d_\chi(q_O, q_i) = d_S(q_O, q_i^*)$.
7. For our purpose, the optimal shape distance is
$$inf_{i=\overline{0,N}} d_\chi(q_O, q_i) = inf_{i=\overline{0,N}} d_S(q_O, q_i^*) = d_S(q_O, q_j^*)$$
such that $0 \le j \le N$ to find the part of $\beta_2$ that gives the best alignment to $\beta_O$, which is $\beta_j^*$ such that $\beta_j^*(t) = \int_0^t q_j^*(s)|q_j^*(s)|\,ds$.
8. Since $j$ is the solution, the starting parameter is $t_j$, and the sub-domain is $[t_j; t_j + c]$.

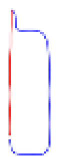
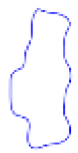
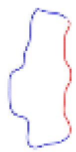
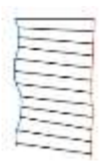
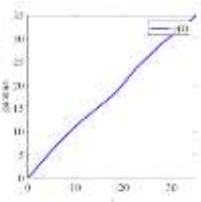
***Table.1*** shows the results of our algorithm applied to two shapes from MPEG-7 , (a) represent $\beta_O$ in red, the target part, within $\beta_1$, $\beta_2$ the closed curve in (b), (c) show the part $\beta_j^*$ in red, from $\beta_2$, that gives the best elastic registration to $\beta_O$, (d) is the best matching between points on parts, (e) gives the geodesic path between them and (f) is the best re-parameterization on $[0; c]$ (here we translate $[t_j; t_j + c]$ to $[0; c]$).

## 4. Experimental Results

In order to estimate the efficiency of the proposed algorithm concluded for partial shape analysis, we apply it using Windows 32 bit PC powered by an Intel 2.6GHz in MATLAB R2020, on shapes from MPEG-7 dataset, and our MATLAB code is based the one in the linked file proposed by A. Srivastava (2020) in [12] We proceed the following steps :

1. Two closed curves are represented by $(2 \times 100)$ matrices.
2. Extract from one of these matrices, a $(2 \times c)$ matrix that represent an open curve, which starting from the first column.
3. On the other matrix, by changing the starting column, in each iteration, we construct a $(2 \times c)$ matrix.
4. By Appling the MATLAB Code from [12], we obtain the shape distance, optimal rotation, and optimal re-presentation in each iteration.
5. The results of these shape distances form a $(1 \times 100 - c)$ matrix, then we indicate the minimum of these values and its rank, which are our goal, to determine $O_j^*$ and $\gamma_j^*$, where $\gamma_j^*$ is plotted on $[0; c]$.

***Table. 1*** *Illustration of the results of our algorithm applied on two different shapes from MPEG-7 Dataset.*

| | | | |
|---|---|---|---|
|  |  |  |  |
| (a) $\beta_o$ in red within $\beta_1$ | (b) $\beta_2$ | (c) $\beta_j^*$ in red within $\beta_2$ | (d) Best matching between points of parts |
|  | | |  |
| (f) Geodesic path between the two parts | | | (e) Optimal re-parameterization |

If the closed curves are the same, we will obtain identical parts and $\gamma_j^*$ represent the identity function.

## 5. Results and Discussions

***Table. 2*** represents more examples from MPEG-7 dataset, where for each example, $\beta_O$ is the target

- .

***Table. 2*** *Examples of part-to-part partial shape matching in the MPEG7 dataset with the proposed method.*

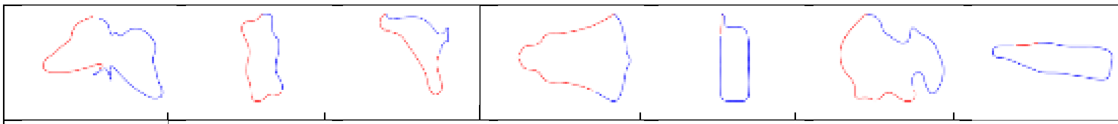
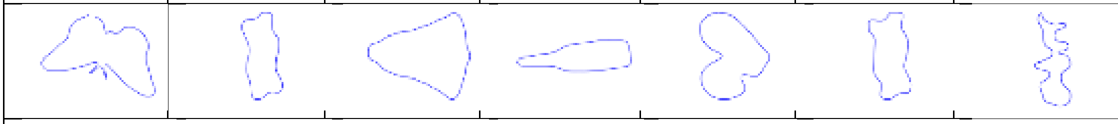
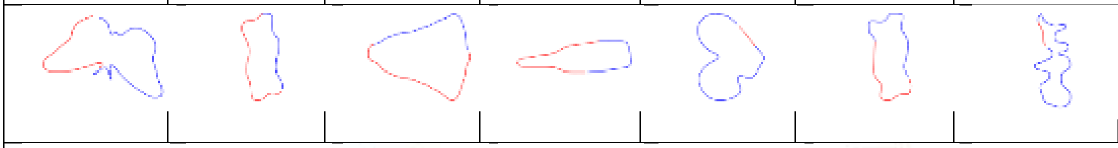
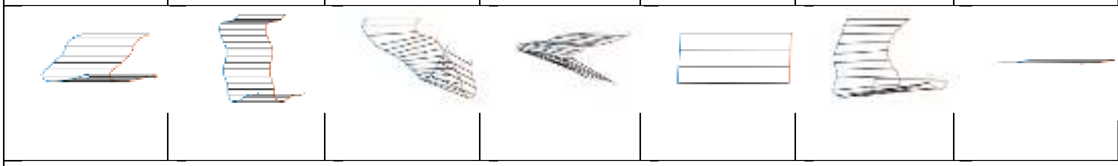
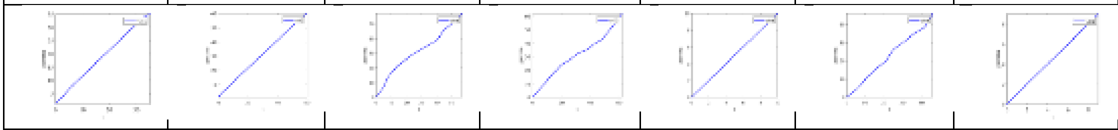
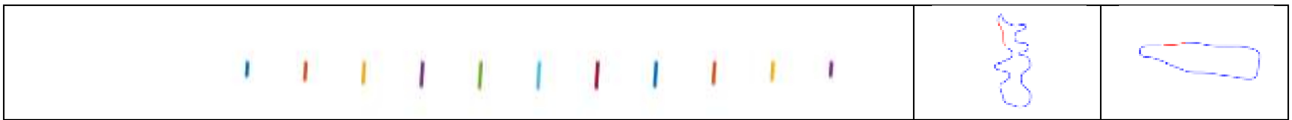
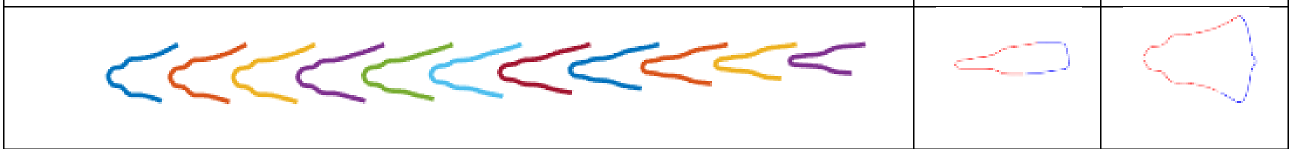
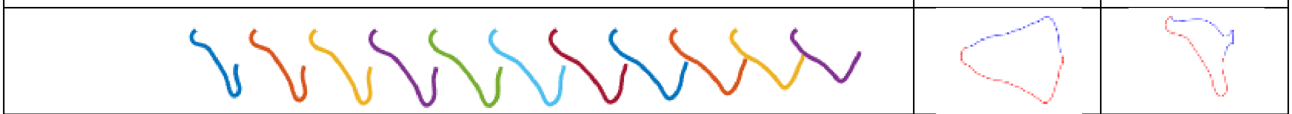
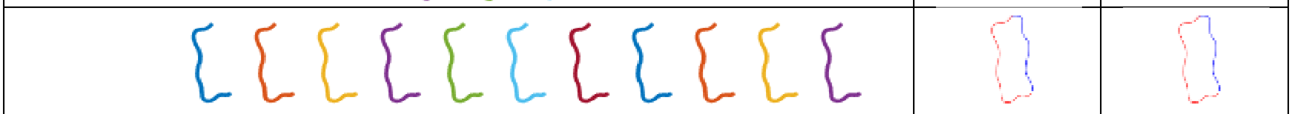
| | | | | | | | |
|---|---|---|---|---|---|---|---|
|  | | | | | | | $\beta_O$ in red within $\beta_1$ |
|  | | | | | | | $\beta_2$ |
|  | | | | | | | $\beta_j^*$ in red within $\beta_2$ |
|  | | | | | | | Best matching between points in parts |
|  | | | | | | | Optimal re-parametrization |
| $d_\chi = 0.092$ | $d_\chi = 0.0711$ | $d_\chi = 1.0329$ | $d_\chi = 0.9548$ | $d_\chi = 1.4694$ | $d_\chi = 1.1338$ | $d_\chi = 1.4764$ | Shape distance |

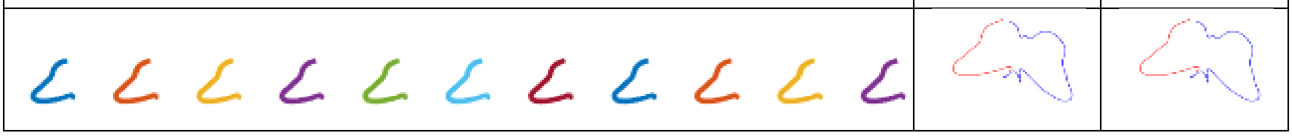
***Table. 3*** *Examples of geodesic paths between the target parts of shapes and the best parts obtained*











part, within the first curve $\beta_1$, and the best part obtained is $\beta_j^*$ from $\beta_2$, by applying our proposed method of elastic partial matching. Then we show the best matching between points of these parts , and the optimal re-parameterization is plotted on $[0; c]$ , if the curves $\beta_1$ and $\beta_2$ are the same, we obtain identical parts $\beta_O$ and $\beta_j^*$, and the re-parameterization function is the identity. Geodesic paths between the parts $\beta_O$ and $\beta_j^*$ in these examples are represented in ***Table. 3.***

## 6. Conclusions

We propose a novel algorithm for elastic partial matching, designed for whole-to-part task. The method finds the best registration for a part of a closed curve (the target) within another closed curve by identifying the segment of equal length that minimizes the Riemannian shape distance across all possible parts with different starting points. The result gives a geodesic deformation path and the optimal re-parameterization function. This framework can be extended to 3D shape analysis in multi-field applications.

## Author Statements:


- **Ethical approval:** The conducted research is not related to either human or animal use.
- **Conflict of interest:**The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper
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**Data availability statement:** The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions


## References


[1] Buchin, K., Buchin, M., & Wang, Y. (2009). Exact algorithms for partial curve matching via the Fréchet distance. In *Proceedings of the twentieth annual ACM-SIAM symposium on Discrete algorithms* (pp. 645–654). Society for Industrial and Applied Mathematics. https://dl.acm.org/doi/10.5555/1496770.1496841

[2] Cui, M., Femiani, J., Hu, J., Wonka, P., & Razdan, A. (2009). Curve matching for open 2D curves. *Pattern Recognition Letters, 30*(1), 1–10. https://doi.org/10.1016/j.patrec.2008.08.013

[3] Funkhouser, T., &Shilane, P. (2006). Partial matching of 3D shapes with priority-driven search. In *Proceedings of the fourth Eurographics symposium on Geometry processing* (pp. 131–142). Eurographics Association. https://dl.acm.org/doi/10.5555/1281957.1281974

[4] Kurtek, S., & Srivastava, A. (2014). Handwriting text segmentation using elastic shape analysis. In *2014 22nd International Conference on Pattern Recognition* (pp. 2501–2506). IEEE. https://doi.org/10.1109/ICPR.2014.432

[5] Latecki, L. J., Megalooikonomou, V., Wang, Q., & Yu, D. (2007). An elastic partial shape matching technique. *Pattern Recognition, 40*(11), 3069–3080. https://doi.org/10.1016/j.patcog.2007.03.004

[6] Maheshwari, A., Sack, J.-R., Shahbaz, K., &Zarrabi-Zadeh, H. (2011). Improved algorithm for partial curve matching. In C. Demetrescu& M. M. Halldórsson (Eds.), *Algorithms – ESA 2011* (Vol. 6942, pp. 518–529). Springer. https://doi.org/10.1007/978-3-642-23719-5_44

[7] Matthews, G. J., Bharath, K., Kurtek, S., Brophy, J. K., Thiruvathukal, G. K., &Harrel, O. (2021). Shape-based classification of partially observed curves, with application to anthropology. *Frontiers in Applied Mathematics and Statistics, 7*, Article 759622. https://doi.org/10.3389/fams.2021.759622

[8] Michel, D., Oikonomidis, I., & Argyros, A. (2011). Scale invariant and deformation tolerant partial shape matching. *Image and Vision Computing, 29*(7), 459–469. https://doi.org/10.1016/j.imavis.2011.01.008

[9] Niels, R., &Vuurpijl, L. (2005). Using dynamic time warping for intuitive handwriting recognition. In *Advances in Graphonomics: Proceedings of the 12th Conference of the International Graphonomics Society* (pp. 217–221). International Graphonomics Society.

[10] Richardson, T., & Wang, S. (2006). Open-curve shape correspondence without endpoint correspondence. In R. Larsen, M. Nielsen, & J. Sporring (Eds.), *Medical Image Computing and Computer-Assisted Intervention – MICCAI 2006* (Vol. 4190, pp. 17–24). Springer. https://doi.org/10.1007/11866565_3

[11] Sebastian, T. B., Klein, P. N., & Kimia, B. B. (2003). On aligning curves. *IEEE Transactions on Pattern Analysis and Machine Intelligence, 25*(1), 116–125. https://doi.org/10.1109/TPAMI.2003.1159951

[12] Srivastava, A. (2020). *Functional and shape data analysis* [Seminar presentation]. TAMIDS Seminar Series, Texas A&M University. https://tamids.tamu.edu/seminar-series/

[13] Srivastava, A., &Klassen, E. P. (2016). *Functional and shape data analysis* (1st ed.). Springer. https://doi.org/10.1007/978-1-4939-4020-2

[14] Srivastava, A., Klassen, E., Joshi, S. H., & Jermyn, I. H. (2011). Shape analysis of elastic curves in Euclidean spaces. *IEEE Transactions on Pattern Analysis and Machine Intelligence, 33*(7), 1415–1428. https://doi.org/10.1109/TPAMI.2010.184

[15] Yang, C., Wei, H., & Yu, Q. (2018). A novel method for 2D nonrigid partial shape matching. *Neurocomputing, 275*, 1160–1176. https://doi.org/10.1016/j.neucom.2017.09.067