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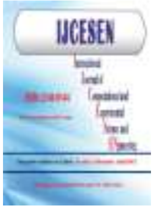

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Research Article

Description of Regular m-Bipolar Fuzzy Graphs

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Abstract:

The first and second basic sequences in an m-BPFG are defined in this work along with an example of an adjacency sequence of a node. A few instances are created to demonstrate that, even if an m-BPFG G is regular, the crisp underlying graph may or may not be regular or have a uniform adjacency sequence for all of the nodes. Additionally, it is demonstrated that not all of the nodes need to have the same adjacency sequence if an m-PPFG G and its crisp underlying graph are regular. The notion of adjacency sequences is used to create a necessary and sufficient requirement for an m-BPFG to be regular if it has no more than four nodes. Along with, there are several definitions of what it means for a regular m-BPFG's complement, line graph, and other properties to be regular.

1. Introduction

Real-world applications of graph theory may be found in an extensive range of areas, such as artificial, quantum intelligence, operations research, indicator processing, system routing, electrical, electronic engineering, information technology, and more. Zadeh [1] substituted the general set with the fuzzy set in 1965, opening a door for study in a variety of domains. Each component in a fuzzy set has a relationship value assigned to it that is chosen from the range $[0,1]$. Bipolar fuzzy sets might be capable for used to more accurately express the vagueness of a set than fuzzy sets, which rely on certain relationship values. This is based on the notion that "bipolar information" is real [2, 3]. More graphical depiction of vague data is made possible by bipolar fuzzy sets, which greatly improves analysis of data interactions, incompleteness, and comparison metrics. Graph theory is a crucial tool for mathematical modeling in addition to being a well-developed area of mathematics. Rosenfeld [4], who initially developed the idea of fuzzy graphs, recognized its significance. While Craine [5] described fuzzy interval graphs and Mordeson and Peng [6] studied numerous activities on fuzzy graphs. Furthermore, Sunitha and Vijaya kumar [7] developed a technique to determine a fuzzy graph's complement, fuzzy hypergraphs are examined by Mordeson and Nair [8], Nair and Cheng [9] presented fuzzy cliques and cliques in fuzzy graphs and studied fuzzy hypergraphs. Nagoorgani and Radha [10] established regularity of a fuzzy graph in 2008. As an extension of a fuzzy graph, Yang et al. [11] introduced general bipolar fuzzy graph in 2013. Bipolar fuzzy hypergraphs were developed by Samanta and Pal [12], complement and isomorphism on BFGs were examined by Talebi and Rashmanlou [13], and extended regular BFG was described by Ghorai and Pal [14-18]. On bipolar fuzzy graphs, Poulik and Ghorai [19-22] established a variety of indices. Numerous studies have been conducted using these BFG principles to far. You may find some of them in [23]. Numerous significant outcomes were examined by Borzooei and Rashmanlou [24-28, 29-31] on vague graphs and bipolar fuzzy graphs.
Research in artificial intelligence and quantum computing makes it necessary to introduced

concepts related to logical causality and decision making integral to technical applications. This can be to some extent captured by the m-BPFG as this theory provides possibility for a ternary classification in the place at conventional binary classification.

We know that we have positive (+) and negative (-) as well-established bipolar coordinates for traditional graph analysis. However, complex developments in the field of science tell us that there are categories which are both positive and negative at the given point of time [32-41].

This can be demonstrated with the example of opinion making behavior of a country like India. Though India has a democratically elected government at the center, each state has its own democratically elected state governments. While opinions regarding many central government policies are expressed as agreements or oppositions by state governments, there are some central policies where there is a degree of both agreement and opposition. Let us explain an example from latest farm law proposed by the government of India with all 29 states taken as an m-block. In principle, this law was directly accepted by 20 states while six states opposed it. There are three states which expressed modifications in the policy as it evolved a mixed response. This is important as it influences the political system and patterns of voting in India.

However, in each of the states that have accepted the policy some sections of the farmers opposed the policy; and in the states which opposed the policy there was a call for agreement from some sections of the farmers.

The first and second basic sequences, as well as the adjacency sequence of a node, are defined in this study. A few instances are created to demonstrate that, even if G is a regular m-BPFG, the crisp underlying graph may or may not be be regular or have a uniform adjacency sequence for all of the nodes. An m-BPFG is formed under some conditions that are both required and sufficient.

2. Preliminaries

The term "m-BPFG" is defined in this section. For the m-BPFG to be generalized, an equivalence criterion was established. Create a relationship of equivalence from the given set V, $\leftrightarrow$ on $V \times V - \{(\iota,\iota): \iota \in V\}$ as follows: $(\iota_1,\kappa_1) \leftrightarrow (\iota_2,\kappa_2)$ if and only if either $(\iota_1,\kappa_1) = (\iota_2,\kappa_2$ or $\iota_1 = \kappa_2, \kappa_1 = \iota_2$.

The equivalence class containing the components (ι,κ) is represented by $\iota\kappa$ or $\kappa\iota$, while the quotient set is represented by $\overleftrightarrow{V^2}$.

Definition 2.1: An m-BPFG of a graph $G^* = (V,E)$ is a pair $G = (V,Q,R)$ where $Q = \langle [P_h\text{o}\Psi_Q^+, P_h\text{o}\Psi_Q^-]_{h=1}^m \rangle$, $P_h\text{o}\Psi_Q^+ : V \to [0,1]$ and $P_h\text{o}\Psi_Q^- : V \to [-1,0]$ is an m-BPFS on V and $R = \langle [P_h\text{o}\Psi_R^+, P_h\text{o}\Psi_R^-]_{h=1}^m \rangle$, $P_h\text{o}\Psi_R^+ : \overleftrightarrow{V^2} \to [0,1]$ and $P_h\text{o}\Psi_Q^- : \overleftrightarrow{V^2} \to [-1,0]$ in an m-BPFS in $\overleftrightarrow{V^2}$ such that

$$P_h\text{o}\Psi_R^+(\iota\kappa) \le \min\{P_h\text{o}\Psi_Q^+(\iota), P_h\text{o}\Psi_Q^+(\kappa)\},$$
$$P_h\text{o}\Psi_R^-(\iota\kappa) \ge \max\{P_h\text{o}\Psi_Q^-(\iota), P_h\text{o}\Psi_Q^-(\kappa)\}$$

for all $\iota\kappa \in \overleftrightarrow{V^2}, h = 1,2,\cdots,m$ and $P_h\text{o}\Psi_R^+(\iota\kappa) = P_h\text{o}\Psi_R^-(\iota\kappa) = 0$ for all $\iota\kappa \in \overleftrightarrow{V^2} - E$.

Definition 2.2: Let $G = (V,Q,R)$ be an m-BPFG of $G^* = (V,E)$. The complement of G is an m-BPFG $\bar{G} = (V,\bar{Q},\bar{R})$ of $\overline{G^*} = (V, \overleftrightarrow{V}^2)$ such that $\bar{Q} = Q$ and $\bar{R} = \langle [P_h\text{o}\Psi_{\bar{R}}^+, P_h\text{o}\Psi_{\bar{R}}^-]_{h=1}^m \rangle$ is defined by

$P_h\text{o}\Psi_{\bar{R}}^+(\iota\kappa) = \min\{P_h\text{o}\Psi_Q^+(\iota), P_h\text{o}\Psi_Q^+(\kappa)\} - P_h\text{o}\Psi_R^+(\iota\kappa)$,

$P_h\text{o}\Psi_{\bar{R}}^-(\iota\kappa) = \max\{P_h\text{o}\Psi_Q^-(\iota), P_h\text{o}\Psi_Q^-(\kappa)\} - P_h\text{o}\Psi_R^-(\iota\kappa)$ for $\iota\kappa \in \overleftrightarrow{V}^2$.

Definition 2.3: Let $G = (V,Q,R)$ be an m-BPFG of $G^* = (V,E)$.

(a) Then, $d_G(\iota)$stands for the open neighborhood degree of a node $\iota \in V$ and is defined as $d_G(\iota) = \langle [P_h\text{od}_G^+(\iota), P_h\text{od}_G^-(\iota)]_{h=1}^m \rangle = \langle \begin{bmatrix} \sum_{\substack{\iota\neq\kappa \\ \iota\kappa\in E}} P_h\text{o}\Psi_R^+(\iota\kappa), \\ \sum_{\substack{\iota\neq\kappa \\ \iota\kappa\in E}} P_h\text{o}\Psi_R^-(\iota\kappa) \end{bmatrix}_{h=1}^m \rangle$.

(b) When all of G's nodes have the same open neighborhood degree, $\langle [\beta_h^+, \beta_h^-]_{h=1}^m \rangle$, G is referred to as $\langle [\beta_h^+, \beta_h^-]_{h=1}^m \rangle$ −regular.

(c) Then, $d_G[\iota]$stands for the closed neighborhood degree of a node $\iota \in V$ and is defined as $d_G[\iota] = \langle [P_h\text{od}_G^+[\iota], P_h\text{od}_G^-[\iota]]_{h=1}^m \rangle = \langle [P_h\text{od}_G^+(\iota) + P_h\text{o}\Psi_Q^+(\iota), P_h\text{od}_G^-(\iota) + P_h\text{o}\Psi_Q^-(\iota)]_{h=1}^m \rangle$.

When all of G 's nodes have the same closed neighborhood degree, $\langle [\gamma_h^+, \gamma_h^-]_{h=1}^m \rangle$, G is referred to as $\langle [\gamma_h^+, \gamma_h^-]_{h=1}^m \rangle$ −totally regular.

3. Adjacency sequences

Below, we'll go through the idea of an adjacency sequence and several basic m-BPFG sequences.

Definition 3.1: Let $L(G^*) = (Z,W)$ be a line graph of a (simple) graph $G^* = (V,E)$.

Let $G = (V,Q,R)$be an m-BPFG of G^*. Then an m-BPFLG $L(G) = (Q_1,R_1)$ of G is defined as follows:

(i) Q_1 and $R_1$are m-BPF subsets of Z and W respectively,

(ii) $P_h\text{o}\Psi^{+}_{Q_1}(S_\iota) = P_h\text{o}\Psi^{+}_{R}(\iota) = P_h\text{o}\Psi^{+}_{R}(q_\iota r_\iota)$, $P_h\text{o}\Psi^{-}_{Q_1}(S_\iota) = P_h\text{o}\Psi^{-}_{R}(\iota) = P_h\text{o}\Psi^{-}_{R}(q_\iota r_\iota)$,

(iii) $P_h\text{o}\Psi^{+}_{R_1}(S_\iota S_\kappa) = \min\{P_h\text{o}\Psi^{+}_{R}(\iota), P_h\text{o}\Psi^{+}_{R}(\kappa)\} = \min\{P_h\text{o}\Psi^{+}_{R}(q_\iota r_\iota), P_h\text{o}\Psi^{+}_{R}(q_\kappa r_\kappa)\}$, $P_h\text{o}\Psi^{-}_{R_1}(S_\iota S_\kappa) = \max\{P_h\text{o}\Psi^{-}_{R}(\iota), P_h\text{o}\Psi^{-}_{R}(\kappa)\} = \max\{P_h\text{o}\Psi^{-}_{R}(q_\iota r_\iota), P_h\text{o}\Psi^{-}_{R}(q_\kappa r_\kappa)\}$ for all $S_\iota, S_\kappa \in Z, S_\iota S_\kappa \in W$ and $h = 1, 2, \cdots, m$.

Definition 3.2: The adjacency sequence of a node τ in an m-BPFG G is represented as $\mathcal{A}_{JCS}(\tau) = \langle [P_h o\mathcal{A}_{JCS}(\tau)^+ ,\ P_h o\mathcal{A}_{JCS}(\tau)^-]_{h=1}^{m} \rangle$, where $P_h o\mathcal{A}_{JCS}(\tau)^+ = [\mathfrak{p}_1, \mathfrak{p}_2, \cdots, \mathfrak{p}_n]$ and $P_h o\mathcal{A}_{JCS}(\tau)^- = [\mathfrak{n}_1, \mathfrak{n}_2, \cdots, \mathfrak{n}_n]$, $h = 1, 2, \cdots, m$ indicate that the edges that are adjacent to τ are sorted in increasing order of their respective positive and negative relationship values.

Example 3.3: Consider an m-BPFG $G = (V, Q, R)$ as shown in the Figure. 1., here

$$Q = \left\{ \frac{\alpha}{\langle[0.5,-0.1],[0.8,-0.9]\rangle}, \frac{\beta}{\langle[0.4,-0.5],[0.6,-0.8]\rangle}, \frac{\gamma}{\langle[0.5,-0.6],[0.6,-0.7]\rangle}, \frac{\iota}{\langle[0.4,-0.5],[0.4,-0.7]\rangle}, \frac{\kappa}{\langle[0.5,-0.4],[0.3,-0.5]\rangle}, \frac{\tau}{\langle[0.1,-0.3],[0.5,-0.7]\rangle} \right.$$

$$R = \left\{ \frac{\alpha\beta}{\langle[0.3,-0.1],[0.5,-0.3]\rangle}, \frac{\alpha\gamma}{\langle[0.2,-0.1],[0.5,-0.6]\rangle}, \frac{\beta\gamma}{\langle[0.3,-0.2],[0.4,-0.5]\rangle}, \frac{\gamma\iota}{\langle[0.2,-0.3],[0.1,-0.5]\rangle}, \frac{\iota\tau}{\langle[0.1,-0.2],[0.3,-0.6]\rangle}, \frac{\tau\kappa}{\langle[0.1,-0.2],[0.1,-0.4]\rangle}, \frac{\iota\kappa}{\langle[0.3,-0.2],[0.2,-0.4]\rangle} \right.$$

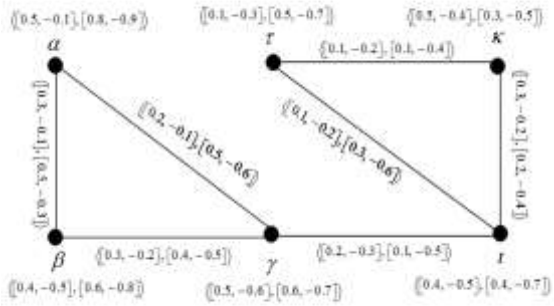

***Figure 1:** m-Bipolar Fuzzy Graph and its Adjacency Sequences*

Then the adjacency sequences of the nodes are

$\mathcal{A}_{JCS}(\alpha) = \langle [P_h o\mathcal{A}_{JCS}(\alpha)^+, P_h o\mathcal{A}_{JCS}(\alpha)^-]_{h=1}^{m} \rangle$
$= \langle [0.2, 0.3], [-0.1, -0.1], [0.5, 0.5], [-0.6, -0.3] \rangle$,

$\mathcal{A}_{JCS}(\beta) = \langle [P_h o\mathcal{A}_{JCS}(\beta)^+, P_h o\mathcal{A}_{JCS}(\beta)^-]_{h=1}^{m} \rangle$
$= \langle [0.3, 0.3], [-0.2, -0.1], [0.4, 0.5], [-0.5, -0.3] \rangle$,

$\mathcal{A}_{JCS}(\gamma) = [P_h o\mathcal{A}_{JCS}(\gamma)^+, P_h o\mathcal{A}_{JCS}(\gamma)^-]$
$= \langle [0.2, 0.2, 0.3], [-0.3, -0.2, -0.1], [0.1, 0.4, 0.5], [-0.6, -0.5, -0.5] \rangle$,

$\mathcal{A}_{JCS}(\iota) = \langle [P_h o\mathcal{A}_{JCS}(\iota)^+, P_h o\mathcal{A}_{JCS}(\iota)^-]_{h=1}^{m} \rangle$
$= \langle [0.1, 0.2, 0.3], [-0.3, -0.2, -0.2], [0.1, 0.2, 0.3], [-0.6, -0.5, -0.4] \rangle$,

$\mathcal{A}_{JCS}(\tau) = \langle [P_h o\mathcal{A}_{JCS}(\tau)^+, P_h o\mathcal{A}_{JCS}(\tau)^-]_{h=1}^{m} \rangle$
$= \langle [0.1, 0.1], [-0.2, -0.2], [0.1, 0.3], [-0.6, -0.4] \rangle$,

$\mathcal{A}_{JCS}(\kappa) = \langle [P_h o\mathcal{A}_{JCS}(\kappa)^+, P_h o\mathcal{A}_{JCS}(\kappa)^-]_{h=1}^{m} \rangle$
$= \langle [0.1, 0.3], [-0.2, -0.2], [0.1, 0.2], [-0.4, -0.4] \rangle$.

Remark 3.4: We now have result in drastic findings.

(i) The number of points in $P_h o\mathcal{A}_{JCS}(\tau)^+$ or $P_h o\mathcal{A}_{JCS}(\tau)^-$ is the degree of τ in G^* for any h.

(ii) The sum of all points in $[P_h o\mathcal{A}_{JCS}(\tau)^+]_{h=1}^{m}$ and sum of all points in $[P_h o\mathcal{A}_{JCS}(\tau)^-]_{h=1}^{m}$ is a degree of τ in an m-BPFG G , i.e. $d_G(\tau) = \langle [\sum_{k_j \in [P_h o\mathcal{A}_{JCS}(\tau)^+]} k_j ,\ \sum_{k_j \in [P_h o\mathcal{A}_{JCS}(\tau)^-]} k_j]_{h=1}^{m} \rangle$.

Remark 3.5: If G is a regular m-BPFG, the crisp underlying graph G^* may not be regular and every node may not contain the equal adjacency sequence.

For instance, Take an m-BPFG G of G^* in Figure. 2., here

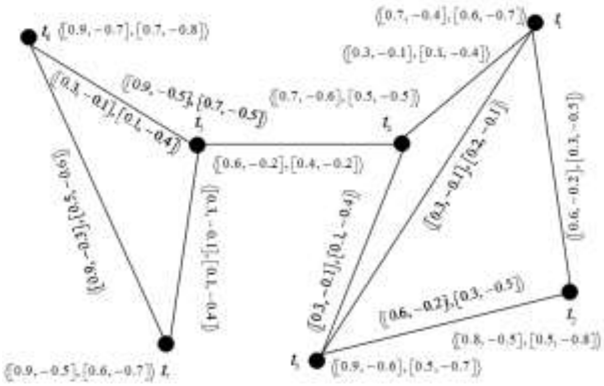

***Figure 2:** G is an m-BPFG and Regular. Where the Nodes in G should not contain the same Adjacency Sequence and G^* is not Regular*

$$Q = \left\{ \frac{\iota_1}{\langle[0.7,-0.4],[0.6,-0.7]\rangle}, \frac{\iota_2}{\langle[0.8,-0.5],[0.5,-0.8]\rangle}, \frac{\iota_3}{\langle[0.9,-0.6],[0.5,-0.7]\rangle}, \frac{\iota_4}{\langle[0.7,-0.6],[0.5,-0.5]\rangle}, \frac{\iota_5}{\langle[0.9,-0.5],[0.7,-0.5]\rangle}, \frac{\iota_6}{\langle[0.9,-0.7],[0.7,-0.8]\rangle}, \frac{\iota_7}{\langle[0.9,-0.5],[0.6,-0.7]\rangle} \right\}$$

$$R = \left\{ \frac{\iota_1\iota_2}{\langle[0.6,-0.2],[0.3,-0.5]\rangle}, \frac{\iota_2\iota_3}{\langle[0.6,-0.2],[0.3,-0.5]\rangle}, \frac{\iota_3\iota_4}{\langle[0.3,-0.1],[0.1,-0.4]\rangle}, \frac{\iota_4\iota_1}{\langle[0.3,-0.1],[0.1,-0.4]\rangle}, \frac{\iota_1\iota_3}{\langle[0.3,-0.1],[0.2,-0.1]\rangle}, \frac{\iota_4\iota_5}{\langle[0.6,-0.2],[0.4,-0.2]\rangle}, \frac{\iota_5\iota_6}{\langle[0.3,-0.1],[0.1,-0.4]\rangle}, \frac{\iota_6\iota_7}{\langle[0.9,-0.3],[0.5,-0.6]\rangle}, \frac{\iota_5\iota_7}{\langle[0.3,-0.1],[0.1,-0.4]\rangle} \right\}.$$

We see that G is $\langle[1.2,-0.4],[0.6,-1]\rangle$-regular m-BPFG whose crisp underlying graph G^* is not regular.
Furthermore,

$$\mathcal{A}_{JCS}(\iota_1) = \langle\begin{matrix}[0.3,0.3,0.6],[-0.2,-0.1,-0.1],,\\ [0.1,0.2,0.3],[-0.5,-0.4,-0.1]\end{matrix}\rangle,$$

$$\neq \mathcal{A}_{JCS}(\iota_2) = \langle\begin{matrix}[0.6,0.6],[-0.2,-0.2,],[0.3,0.3],\\ [-0.5,-0.5]\end{matrix}\rangle.$$

Therefore, it is not necessary for every node in a regular m-BPFG to have the identical adjacency sequence.

Remark 3.6: It is possible that certain nodes do not have the same adjacency sequence if G and G^* are both regular.

Definition 3.7: The basic sequence of an m-BPFG G is characterized as the structured set $\xi_{bs}(G) = \langle[P_h o\xi_{bs}^+(G),\ P_h o\xi_{bs}^-(G)]_{h=1}^m\rangle$ where $P_h o\xi_{bs}^+(G) = \{P_h o\Psi_Q^+(\iota) > 0: \iota \in V\} \cup \{P_h o\Psi_R^+(\iota\kappa) > 0: \iota,\kappa \in V\}$, $P_h o\xi_{bs}^-(G) = \{P_h o\Psi_Q^-(\iota) < 0: \iota \in V\} \cup \{P_h o\Psi_R^-(\iota\kappa) < 0: \iota,\kappa \in V\}$, for $h = 1,2,\cdots,m$. $P_h o\xi_{bs}^+(G)$ and $P_h o\xi_{bs}^-(G)$ have elements that are arranged in decreasing or increasing order, respectively.

The first basic sequence is known as the node portion of the basic sequence, while the second basic sequence is known as the edge part of the basic sequence. These parts are indicated by the letters $f\xi_{bs}(G)$ and $s\xi_{bs}(G)$ respectively.

4. Description of regular m-BPFGs

The regular m-BPFGs which have four nodes or less are outlined by the theorem.

Theorem 4.1: Let k be the number of nodes in G and let $G = (V,Q,R)$ be an m-BPFG of $G^* = (V,E)$. G is a regular m-BPFG iff G^* is regular and every node has the equal adjacency sequence, which holds true if $k \leq 4$.

Proof: Suppose that G^* is regular and every node has the equal adjacency sequence. Thus G is a regular m-BPFG.

On the other hand, let G be a $\beta = \langle[\beta_h^+,\beta_h^-]_{h=1}^m\rangle$ −regular m-BPFG.

There is nothing to prove if there is only one node or if there is no edge connecting the nodes. Thus, we presume that $E \neq \emptyset$. Then $k > 1$and $\beta \neq 0$.Think of the three scenarios when $k = 2,3$ and 4.

Case 1: If $k = 2$, G contains two nodes ι_1 and ι_2. Hence G^* is one regular and ι_1 and $\iota_2$have the equal adjacency sequences $\mathcal{A}_{JCS}(\iota_1) = \langle[P_h o\Psi_R^+(\iota_1\iota_2)P_h o\Psi_R^-(\iota_1\iota_2)]_{h=1}^m\rangle$.

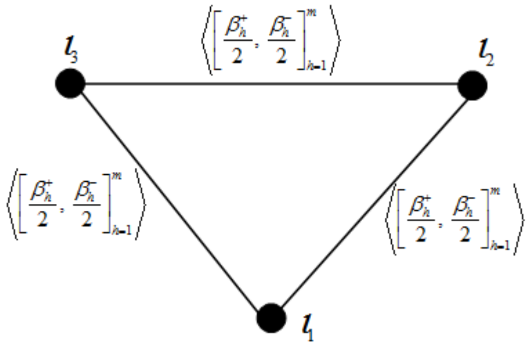

Figure 3.: *G is a $\langle[\beta_h^+,\beta_h^-]_{h=1}^m\rangle$ −Regular m-BPFG with G^* cyclic and 2-Regular*

Case 2: When $k = 3$, G have three nodes ι_1, ι_2, ι_3 with the edges $\iota_1\iota_2, \iota_2\iota_3, \iota_1\iota_3$ in G (see Figure. 3.). Let the edge's relationship values be $\langle[P_h o\Psi_R^+(\iota_1\iota_2)P_h o\Psi_R^-(\iota_1\iota_2)]_{h=1}^m\rangle$, $\langle[P_h o\Psi_R^+(\iota_2\iota_3)P_h o\Psi_R^-(\iota_2\iota_3)]_{h=1}^m\rangle$ and $\langle[P_h o\Psi_R^+(\iota_1\iota_3)P_h o\Psi_R^-(\iota_1\iota_3)]_{h=1}^m\rangle$ respectively.

As G is $\beta = \langle[\beta_h^+,\beta_h^-]_{h=1}^m\rangle$ −regular m-BPFG, $\beta(\iota_j) = \langle[\beta_h^+,\beta_h^-]_{h=1}^m\rangle$ for $j = 1,2,3$.

So we get,

$P_h o\Psi_R^+(\iota_1\iota_2) + P_h o\Psi_R^+(\iota_2\iota_3) = \beta_h^+$ (1)

$P_h o\Psi_R^+(\iota_2\iota_3) + P_h o\Psi_R^+(\iota_1\iota_3) = \beta_h^+$ (2)

$P_h o\Psi_R^+(\iota_1\iota_2) + P_h o\Psi_R^+(\iota_1\iota_3) = \beta_h^+$ (3)

$P_h o\Psi_R^-(\iota_1\iota_2) + P_h o\Psi_R^-(\iota_2\iota_3) = \beta_h^-$ (4)

$P_h o\Psi_R^-(\iota_2\iota_3) + P_h o\Psi_R^-(\iota_1\iota_3) = \beta_h^-$ (5)

$P_h o\Psi_R^-(\iota_1\iota_2) + P_h o\Psi_R^-(\iota_1\iota_3) = \beta_h^-$ (6)

By condensing the equations above, we obtain $P_h o\Psi_R^+(\iota_1\iota_2) = P_h o\Psi_R^+(\iota_2\iota_3) = P_h o\Psi_R^+(\iota_1\iota_3) = \frac{\beta_h^+}{2}$ and $P_h o\Psi_R^-(\iota_1\iota_2) = P_h o\Psi_R^-(\iota_2\iota_3) = P_h o\Psi_R^-(\iota_1\iota_3) = \frac{\beta_h^-}{2}$ for $h = 1,2,\cdots,m$.

So every node has equal adjacency sequence, i.e. $\mathcal{A}_{JCS}(\iota_j) = \langle\left[\left[\frac{\beta_h^+}{2},\frac{\beta_h^+}{2}\right],\left[\frac{\beta_h^-}{2},\frac{\beta_h^-}{2}\right]\right]_{h=1}^m\rangle$, for $j = 1,2,3$ and G^* is two regular. Now G^* is a cycle and every edge contains the relationship value $\langle\left[\frac{\beta_h^+}{2},\frac{\beta_h^-}{2}\right]_{h=1}^m\rangle$.

Case 3: When $k = 4$, G have four nodes ι_1, ι_2, ι_3 , ι_4 with the edges $\iota_1\iota_2, \iota_1\iota_4, \iota_1\iota_3, \iota_2\iota_3, \iota_2\iota_4$ and $\iota_3\iota_4$ in G (see Figure 4.). Let the edge's relationship values be $\langle[P_h o\Psi_R^+(\iota_1\iota_2)P_h o\Psi_R^-(\iota_1\iota_2)]_{h=1}^m\rangle$, $\langle[P_h o\Psi_R^+(\iota_1\iota_4)P_h o\Psi_R^-(\iota_1\iota_4)]_{h=1}^m\rangle$, $\langle[P_h o\Psi_R^+(\iota_1\iota_3)P_h o\Psi_R^-(\iota_1\iota_3)]_{h=1}^m\rangle$, $\langle[P_h o\Psi_R^+(\iota_2\iota_3)P_h o\Psi_R^-(\iota_2\iota_3)]_{h=1}^m\rangle$, $\langle[P_h o\Psi_R^+(\iota_2\iota_4)P_h o\Psi_R^-(\iota_2\iota_4)]_{h=1}^m\rangle$ and $\langle[P_h o\Psi_R^+(\iota_3\iota_4)P_h o\Psi_R^-(\iota_3\iota_4)]_{h=1}^m\rangle$, respectively.

As G is $\beta = \langle[\beta_h^+,\beta_h^-]_{h=1}^m\rangle$ −regular m-BPFG, $\beta(\iota_j) = \langle[\beta_h^+,\beta_h^-]_{h=1}^m\rangle$ for $j = 1,2,3,4$.

So we get,

$P_h o\Psi_R^+(\iota_1\iota_2) + P_h o\Psi_R^+(\iota_1\iota_3) + P_h o\Psi_R^+(\iota_1\iota_4) = \beta_h^+$ (7)

$P_h o\Psi_R^+(\iota_2\iota_1) + P_h o\Psi_R^+(\iota_2\iota_3) + P_h o\Psi_R^+(\iota_2\iota_4) = \beta_h^+$
(8)
$P_h o\Psi_R^+(\iota_3\iota_1) + P_h o\Psi_R^+(\iota_3\iota_2) + P_h o\Psi_R^+(\iota_3\iota_4) = \beta_h^+$
(9)
$P_h o\Psi_R^+(\iota_4\iota_1) + P_h o\Psi_R^+(\iota_4\iota_2) + P_h o\Psi_R^+(\iota_4\iota_3) = \beta_h^+$
(10)
$P_h o\Psi_R^-(\iota_1\iota_2) + P_h o\Psi_R^-(\iota_1\iota_3) + P_h o\Psi_R^-(\iota_1\iota_4) = \beta_h^-$
(11)
$P_h o\Psi_R^-(\iota_2\iota_1) + P_h o\Psi_R^-(\iota_2\iota_3) + P_h o\Psi_R^-(\iota_2\iota_4) = \beta_h^-$
(12)
$P_h o\Psi_R^-(\iota_3\iota_1) + P_h o\Psi_R^-(\iota_3\iota_2) + P_h o\Psi_R^-(\iota_3\iota_4) = \beta_h^-$
(13)
$P_h o\Psi_R^-(\iota_4\iota_1) + P_h o\Psi_R^-(\iota_4\iota_2) + P_h o\Psi_R^-(\iota_4\iota_3) = \beta_h^-$
(14)

By condensing the equations above, we obtain $P_h o\Psi_R^+(\iota_1\iota_3) = P_h o\Psi_R^+(\iota_2\iota_4), P_h o\Psi_R^+(\iota_1\iota_2) = P_h o\Psi_R^+(\iota_3\iota_4), P_h o\Psi_R^+(\iota_1\iota_4) = P_h o\Psi_R^+(\iota_2\iota_3)$ and $P_h o\Psi_R^-(\iota_1\iota_3) = P_h o\Psi_R^-(\iota_2\iota_4),\ P_h o\Psi_R^-(\iota_1\iota_2) = P_h o\Psi_R^-(\iota_3\iota_4),\ P_h o\Psi_R^-(\iota_1\iota_4) = P_h o\Psi_R^-(\iota_2\iota_3)$.

Let $P_h o\Psi_R^+(\iota_1\iota_3) = P_h o\Psi_R^+(\iota_2\iota_4) = \alpha_h, P_h o\Psi_R^+(\iota_1\iota_2) = P_h o\Psi_R^+(\iota_3\iota_4) = \beta_h$, $P_h o\Psi_R^+(\iota_1\iota_4) = P_h o\Psi_R^+(\iota_2\iota_3) = \gamma_h$ and $P_h o\Psi_R^-(\iota_1\iota_3) = P_h o\Psi_R^-(\iota_2\iota_4) = \delta_h, P_h o\Psi_R^-(\iota_1\iota_2) = P_h o\Psi_R^-(\iota_3\iota_4) = \theta_h,\ \ P_h o\Psi_R^-(\iota_1\iota_4) = P_h o\Psi_R^-(\iota_2\iota_3) = \rho_h$.

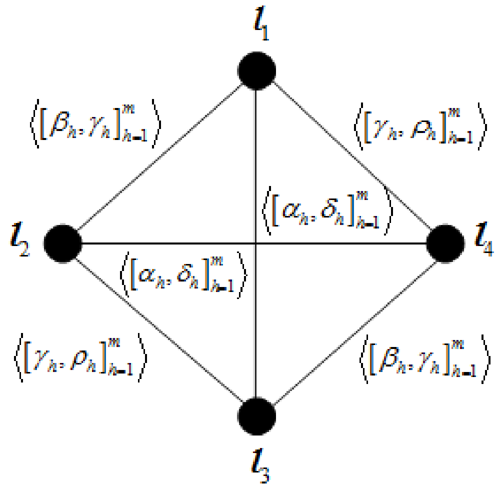

Figure 4.: *G is a $\langle[\beta_h^+, \beta_h^-]_{h=1}^m\rangle$ −Regular m-BPFG with G^* Three-Regular or Two-Regular or One-Regular*

As E is not empty, we have at least one of the edge's relationship values must be nonzero.

If every edge's relationship value is not equals to zero, then G^* is a three-regular graph, the elements in each one node's adjacency sequence are$\langle[[\alpha_h, \beta_h, \gamma_h], [\delta_h, \theta_h, \rho_h]]_{h=1}^m\rangle$in ascending order.

If any two edge's relationship values are not equals to zero, thus G^* is two-regular and each node's adjacency sequence contains the two non-zero relationship values in ascending order.

If anyone edge relationship values is not equals to zero, thus G^* is one-regular and each node's adjacency sequence contains non-zero relationship value.

Remark 4.2: Let $G = (V, Q, R)$ be an m-BPFG of the graph G^* where $V = \{\iota_1, \iota_2, \iota_3, \iota_4, \iota_5\}$ and $E = \{\iota_1\iota_2, \iota_2\iota_3, \iota_3\iota_4, \iota_3\iota_5, \iota_4\iota_5, \iota_5\iota_1\}$ (see Figure 5.). Here $d_G(\iota_i) = \langle[0.9, -0.6], [0.8, -0.7]\rangle$ for all $i = 1, 2, 3, 4, 5$. So G is $\langle[0.9, -0.6], [0.8, -0.7]\rangle$-regular m-BPFG. Hence from the Figure 5., it is shown that G^* is not regular. Again, the adjacency sequences of the nodes are distinct as mentioned under:

Here, $Q =$

$$\left\{\begin{matrix} \frac{\iota_1}{\langle[0.7,-0.5],[0.6,-0.7]\rangle}, \frac{\iota_2}{\langle[0.8,-0.6],[0.7,-0.6]\rangle}, \\ \frac{\iota_3}{\langle[0.8,-0.5],[0.8,-0.7]\rangle}, \\ \frac{\iota_4}{\langle[0.7,-0.6],[0.6,-0.5]\rangle}, \frac{\iota_5}{\langle[0.6,-0.5],[0.8,-0.6]\rangle} \end{matrix}\right\},$$

$$R = \left\{\begin{matrix} \frac{\iota_1\iota_2}{\langle[0.6,-0.4],[0.5,-0.5]\rangle}, \frac{\iota_2\iota_3}{\langle[0.3,-0.2],[0.3,-0.2]\rangle}, \\ \frac{\iota_3\iota_4}{\langle[0.45,-0.3],[0.4,-0.35]\rangle}, \frac{\iota_5\iota_1}{\langle[0.3,-0.2],[0.3,-0.2]\rangle}, \\ \frac{\iota_3\iota_5}{\langle[0.15,-0.1],[0.1,-0.15]\rangle}, \frac{\iota_4\iota_5}{\langle[0.45,-0.3],[0.4,-0.35]\rangle} \end{matrix}\right\},$$

$$\mathcal{A}_{JCS}(\iota_1) = \langle{[0.3,0.6], [-0.4,-0.2], [0.3,0.5], \atop [-0.5,-0.2]}\rangle,$$

$$\mathcal{A}_{JCS}(\iota_2) = \langle{[0.3,0.6], [-0.4,-0.2,], [0.3,0.5], \atop [-0.5,-0.2]}\rangle,$$

$$\mathcal{A}_{JCS}(\iota_3) = \langle{[0.15,0.3,0.45], [-0.3,-0.2,-0.1], \atop [0.1,0.3,0.4], [-0.35,-0.2,-0.15]}\rangle,$$

$$\mathcal{A}_{JCS}(\iota_4) = \langle{[0.45,0.45], [-0.3,-0.3,], [0.4,0.4], \atop [-0.35,-0.35]}\rangle,$$

$$\mathcal{A}_{JCS}(\iota_5) = \langle{[0.15,0.3,0.45], [-0.3,-0.2,-0.1], \atop [0.1,0.3,0.4], [-0.35,-0.2,-0.15]}\rangle,$$

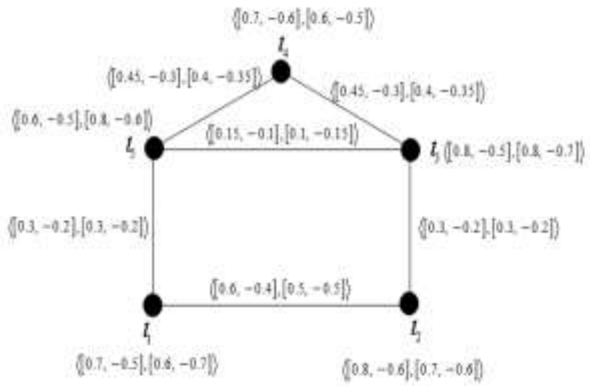

Figure 5.: *G is a $\langle[0.9, -0.6], [0.8, -0.7]\rangle$-Regular however G^* is not regular with distinct adjacency sequences*

Remark 4.3: A regular crisp graph's line graph is always regular. However, with m-BPFG, this finding is no longer valid. A regular m-BPFG need not have a regular m-BPFLG.

For example, consider an m-BPFG $G = (V, Q, R)$ in Figure. 6., here

$$Q = \left\{ \frac{\iota_1}{\langle[0.7,-0.6],[0.5,-0.6]\rangle}, \frac{\iota_2}{\langle[0.6,-0.6],[0.5,-0.5]\rangle}, \frac{\iota_3}{\langle[0.6,-0.7],[0.6,-0.5]\rangle}, \frac{\iota_4}{\langle[0.7,-0.6],[0.5,-0.6]\rangle} \right\}$$

$$R = \left\{ \frac{\iota_1\iota_2}{\langle[0.1,-0.1],[0.3,-0.3]\rangle}, \frac{\iota_1\iota_3}{\langle[0.5,-0.5],[0.2,-0.2]\rangle}, \frac{\iota_1\iota_4}{\langle[0.2,-0.2],[0.1,-0.1]\rangle}, \frac{\iota_2\iota_3}{\langle[0.2,-0.2],[0.1,-0.1]\rangle}, \frac{\iota_2\iota_4}{\langle[0.5,-0.5],[0.2,-0.2]\rangle}, \frac{\iota_3\iota_4}{\langle[0.1,-0.1],[0.3,-0.3]\rangle} \right\}$$

Here $d_G(\iota_i) = \langle[0.8,-0.8],[0.6,-0.6]\rangle$ for all $i = 1,2,3,4$.

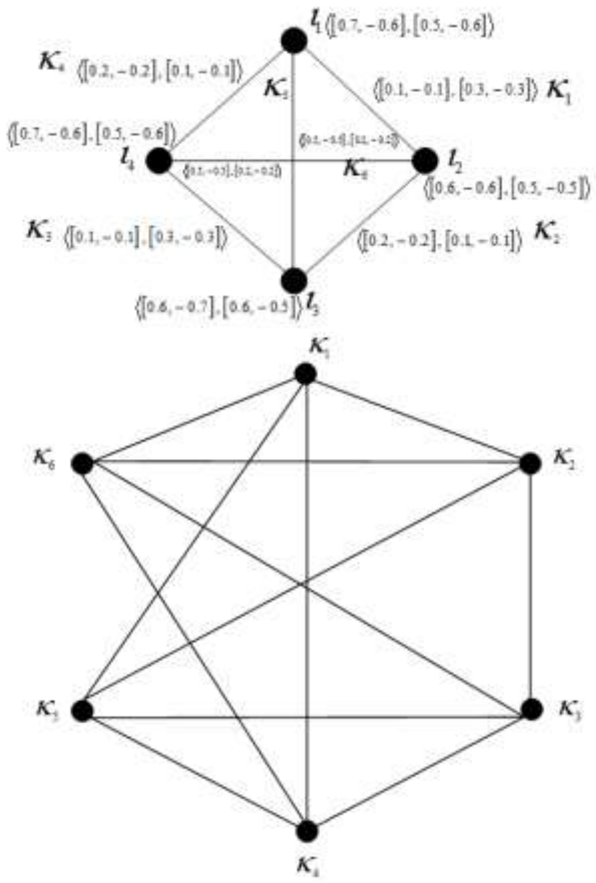

Figure 6: *G is a $\langle[0.8,-0.8],[0.6,-0.6]\rangle$-Regular but the resultant m-BPFLG $L(G)$ is not Regular*

Then, $\mathcal{A}_{JCS}(\iota_1) = \langle\begin{matrix}[0.1,0.2,0.5],[-0.5,-0.2,-0.1],\\ [0.1,0.2,0.3],[-0.3,-0.2,-0.1]\end{matrix}\rangle$,

$\mathcal{A}_{JCS}(\iota_2) = \langle\begin{matrix}[0.1,0.2,0.5],[-0.5,-0.2,-0.1],\\ [0.1,0.2,0.3],[-0.3,-0.2,-0.1]\end{matrix}\rangle$,

$\mathcal{A}_{JCS}(\iota_3) = \langle\begin{matrix}[0.1,0.2,0.5],[-0.5,-0.2,-0.1],\\ [0.1,0.2,0.3],[-0.3,-0.2,-0.1]\end{matrix}\rangle$,

$\mathcal{A}_{JCS}(\iota_4) = \langle\begin{matrix}[0.1,0.2,0.5],[-0.5,-0.2,-0.1],\\ [0.1,0.2,0.3],[-0.3,-0.2,-0.1]\end{matrix}\rangle$,

So G is $\langle[0.8,-0.8],[0.6,-0.6]\rangle$-regular. The relationship values in m-BPFLG are

$$\left\{ \frac{\kappa_1}{\langle[0.1,-0.1],[0.3,-0.3]\rangle}, \frac{\kappa_2}{\langle[0.2,-0.2],[0.1,-0.1]\rangle}, \frac{\kappa_3}{\langle[0.1,-0.1],[0.3,-0.3]\rangle}, \frac{\kappa_4}{\langle[0.2,-0.2],[0.1,-0.1]\rangle}, \frac{\kappa_5}{\langle[0.5,-0.5],[0.2,-0.2]\rangle}, \frac{\kappa_6}{\langle[0.5,-0.5],[0.2,-0.2]\rangle} \right\}$$

$$\left\{ \begin{matrix} \frac{\kappa_1\kappa_2}{\langle[0.1,-0.1],[0.1,-0.1]\rangle}, \frac{\kappa_1\kappa_4}{\langle[0.1,-0.1],[0.1,-0.1]\rangle}, \\ \frac{\kappa_1\kappa_5}{\langle[0.1,-0.1],[0.2,-0.2]\rangle}, \frac{\kappa_1\kappa_6}{\langle[0.1,-0.1],[0.2,-0.2]\rangle}, \\ \frac{\kappa_2\kappa_3}{\langle[0.1,-0.1],[0.1,-0.1]\rangle}, \frac{\kappa_2\kappa_5}{\langle[0.2,-0.2],[0.1,-0.1]\rangle}, \\ \frac{\kappa_2\kappa_6}{\langle[0.2,-0.2],[0.1,-0.1]\rangle}, \frac{\kappa_3\kappa_4}{\langle[0.1,-0.1],[0.1,-0.1]\rangle}, \\ \frac{\kappa_3\kappa_5}{\langle[0.1,-0.1],[0.2,-0.2]\rangle}, \frac{\kappa_3\kappa_6}{\langle[0.1,-0.1],[0.2,-0.2]\rangle}, \\ \frac{\kappa_4\kappa_5}{\langle[0.2,-0.2],[0.1,-0.1]\rangle}, \frac{\kappa_4\kappa_6}{\langle[0.2,-0.2],[0.1,-0.1]\rangle} \end{matrix} \right\}$$

But $d_{L(G)}(\kappa_1) = \langle[0.4,-0.4],[0.6,-0.6]\rangle \neq \langle[0.6,-0.6],[0.4,-0.4]\rangle = d_{L(G)}(\kappa_2)$.

$\mathcal{A}_{JCS}(\kappa_1) = \langle\begin{matrix}[0.1,0.2,0.5],[-0.5,-0.2,-0.1],\\ [0.1,0.2,0.3],[-0.3,-0.2,-0.1]\end{matrix}\rangle$

So the line graph $L(G)$ is not regular.

So, a characterization is shown for a m-BPFLG of a regular m-BPFG to be regular.

Theorem 4.4: Let $G = (V,Q,R)$ be a $\langle[\beta_h^+,\beta_h^-]_{h=1}^m\rangle$- regular m-BPFG of the $k-$ crisp regular graph $G^*$and every node contains the same adjacency sequence $\langle\left[\left[\iota_{1_h}^+,\iota_{2_h}^+,\cdots,\iota_{k_h}^+\right],\left[m_{1_h}^-,m_{2_h}^-,\cdots,m_{k_h}^-\right]\right]_{h=1}^m\rangle$, $\iota_{1_h}^+,\iota_{2_h}^+,\cdots,\iota_{k_h}^+ \in [0,1]$ and $m_{1_h}^-,m_{2_h}^-,\cdots,m_{k_h}^- \in [-1,0]$. Then the m-BPFLG $L(G)$ of G is regular iff R is constant or R takes exactly 3 values so $\iota_{1_h}^+ = \iota_{2_h}^+ = \cdots = \iota_{k-1_h}^+$, $\iota_{j_h}^+ < \iota_{k_h}^+ \leq 1$ for $j = 1,2,\cdots,k-1$ and $m_{2_h}^- = m_{3_h}^- = m_{k-1_h}^- = m_{k_h}^-$, $-1 \leq m_{1_h}^- < m_{j_h}^-$ for $j = 1,2,\cdots,k-1$, $h = 1,2,\cdots,m$.

Proof: Suppose that line graph $L(G)$ is $\langle[\gamma_h^+,\gamma_h^-]_{h=1}^m\rangle$-regular.

Let $R(g) = \langle\left[\iota_{j_h}^+,m_{j_h}^-\right]_{h=1}^m\rangle$, for $j = 1,2,\cdots,k$ where $g = \iota\tau$ be any edge of G.

Thus every ι and τ is incident with $k-1$ edges with relationship values $\left[\iota_{1_h}^+,m_{1_h}^-\right],\left[\iota_{2_h}^+,m_{2_h}^-\right],\cdots,\left[\iota_{j-1_h}^+,m_{j-1_h}^-\right]$, $\left[\iota_{j+1_h}^+,m_{j+1_h}^-\right],\cdots,\left[\iota_{k_h}^+,m_{k_h}^-\right]$. Hence the node g in $L(G)$ is adjacent with $2(k-1)$ edges with relationship values $\left[\iota_{1_h}^+ \wedge \iota_{j_h}^+, m_{1_h}^- \vee m_{j_h}^-\right]$, $\left[\iota_{2_h}^+ \wedge \iota_{j_h}^+, m_{2_h}^- \vee m_{j_h}^-\right]$, $\cdots,\left[\iota_{j-1_h}^+ \wedge \iota_{j_h}^+, m_{j-1_h}^- \vee m_{j_h}^-\right]$, $\left[\iota_{j+1_h}^+ \wedge \iota_{j_h}^+, m_{j+1}^- \vee m_{j_h}^-\right]$, $\cdots,\left[\iota_{k_h}^+ \wedge \iota_{j_h}^+, m_{k_h}^- \vee m_{j_h}^-\right]$, each appearing twice for $h = 1,2,\cdots,m$.

Hence, $d_{L(G)}(g) = \langle [P_h od^+_{L(G)}(g), P_h od^-_{L(G)}(g)]_{h=1}^m \rangle$ where

$$P_h od^+_{L(G)}(g) = 2\sum_{r\neq j} \iota^+_{r_h} \wedge \iota^+_{j_h}$$

$$= 2\sum_{r=1}^{j-1} \iota^+_{r_h} \wedge \iota^+_{j_h} + 2\sum_{r=j+1}^{k} \iota^+_{r_h} \wedge \iota^+_{j_h}$$

$= 2\sum_{r=1}^{j-1} \iota^+_{r_h} + 2\sum_{r=j+1}^{k} \iota^+_{j_h}$ (Since $\iota^+_{1_h} \leq \iota^+_{2_h} \leq \cdots \leq \iota^+_{k_h}$)

$=2\sum_{r=1}^{j-1} \iota^+_{r_h} + 2(k-j)\,\iota^+_{j_h}$

and

$$P_h od^-_{L(G)}(g) = 2\sum_{r\neq j} m^-_{r_h} \vee m^-_{j_h}$$

$$= 2\sum_{r=1}^{j-1} m^-_{r_h} \vee m^-_{j_h} + 2\sum_{r=j+1}^{k} m^-_{r_h} \vee m^-_{j_h}$$

$= 2\sum_{r=1}^{j-1} m^-_{r_h} + 2\sum_{r=j+1}^{k} m^-_{j_h}$

(Since $m^-_{1_h} \leq m^-_{2_h} \leq \cdots \leq m^-_{k_h}$)

$=2(j-1)m^-_{j_h} + 2\sum_{r=j+1}^{k} m^-_{r_h}$.

It is true for each edge g of G.

As $L(G)$ is $\langle [\gamma^+_h, \gamma^-_h]_{h=1}^m \rangle$-regular, $d_{L(G)}(g) = \langle [\gamma^+_h, \gamma^-_h]_{h=1}^m \rangle$-for each node set g in $L(G)$.

So, $2\sum_{r=1}^{j-1} \iota^+_{r_h} + 2(k-j)\,\iota^+_{j_h} = \gamma^+_h$ and $2(j-1)m^-_{j_h} + 2\sum_{r=j+1}^{k} m^-_{r_h} = \gamma^-_h$

i.e. $\sum_{r=1}^{j-1} \iota^+_{r_h} + (k-j)\,\iota^+_{j_h} = \frac{\gamma^+_h}{2}$ and $(j-1)m^-_{j_h} + \sum_{r=j+1}^{k} m^-_{r_h} = \frac{\gamma^-_h}{2}$ for all $j = 1, 2, \cdots, k$ (15)

For $j = 1, 2, \cdots, k$, we get Eq. (15),

$(k-1)\iota^+_{1_h} = \frac{\gamma^+_h}{2}$ and $\sum_{r=2}^{k} m^-_{r_h} = \frac{\gamma^-_h}{2}$ (16)

$\iota^+_{1_h} + (k-2)\iota^+_{2_h} = \frac{\gamma^+_h}{2}$ and $m^-_{2_h} + \sum_{r=3}^{k} m^-_{r_h} = \frac{\gamma^-_h}{2}$ (17)

$\iota^+_{1_h} + \iota^+_{2_h} + (k-3)\iota^+_{3_h} = \frac{\gamma^+_h}{2}$ and $2m^-_{3_h} + \sum_{r=4}^{k} m^-_{r_h} = \frac{\gamma^-_h}{2}$

$\cdots \quad \cdots$

$\cdots \quad \cdots$ (18)

$\iota^+_{1_h} + \iota^+_{2_h} + \cdots + \iota^+_{k-2_h} + \iota^+_{k-1_h} = \frac{\gamma^+_h}{2}$ and $(k-2)m^-_{k-1_h} + m^-_{k_h} = \frac{\gamma^-_h}{2}$ (19)

$\iota^+_{1_h} + \iota^+_{2_h} + \cdots + \iota^+_{k-1_h} = \frac{\gamma^+_h}{2}$ and $(k-1)m^-_{k_h} = \frac{\gamma^-_h}{2}$. (20)

Making It Simpler the above Eqs. (16)-(20), we get $\iota^+_{1_h} = \iota^+_{2_h} = \cdots = \iota^+_{k-2_h} = \iota^+_{k-1_h}$ and $m^-_{2_h} = m^-_{3_h} = \cdots = m^-_{k-1_h} = m^-_{k_h}$.

If $\iota^+_{1_h} = \iota^+_{k_h}$ and $m^-_{1_h} = m^-_{k_h}$, then R is constant otherwise R has three values such that $\iota^+_{1_h} = \iota^+_{2_h} = \cdots = \iota^+_{k-2_h} = \iota^+_{k-1_h}$ and $m^-_{2_h} = m^-_{3_h} = \cdots = m^-_{k-1_h} = m^-_{k_h}$.

Conversely, let R be constant or R contains exactly 3 values so that $\iota^+_{1_h} = \iota^+_{2_h} = \cdots = \iota^+_{k-1_h}, \iota^+_{j_h} < \iota^+_{k_h} \leq 1$ for $j = 1, 2, \cdots, k-1$ and $m^-_{2_h} = m^-_{3_h} = m^-_{k-1_h} = m^-_{k_h}, -1 \leq m^-_{1_h} < m^-_{j_h}$ for $j = 1, 2, \cdots, k-1, h = 1, 2, \cdots, m$.

If R is constant, let $R(\iota\tau) = \langle [\rho^+_h, \rho^-_h]_{h=1}^m \rangle$ for all $\iota\tau \in E$.

Then $\rho^+_h = \iota^+_{j_h}$ and $\rho^-_h = m^-_{j_h}$ for all $j = 1, 2, \cdots, k$.

Therefore, $\iota^+_{r_h} \wedge \iota^+_{j_h} = \rho^+_h$ and $m^-_{r_h} \vee m^-_{j_h} = \rho^-_h$ for all $r \neq j$.

Otherwise, let $\iota^+_{j_h} = \rho^+_h$ for all $j = 1, 2, \cdots, k-1$ and $m^-_{j_h} = \rho^-_h$ for all $j = 1, 2, \cdots, k$.

Then $\iota^+_{j_h} = \rho^+_h < \iota^+_{k_h}$ and $m^-_{1_h} < m^-_{j_h} = \rho^-_h$.

So, $\iota^+_{r_h} \wedge \iota^+_{j_h} = \rho^+_h$ and $m^-_{r_h} \vee m^-_{j_h} = \rho^-_h$ for all $r \neq j$.

So, in every case $\iota^+_{r_h} \wedge \iota^+_{j_h} = \rho^+_h$ and $m^-_{r_h} \vee m^-_{j_h} = \rho^-_h$ for all $r \neq j$.

So, for any node $g = \iota\tau \in L(G)$, we get $d_{L(G)}(g) = \langle [P_h od^+_{L(G)}(g), P_h od^-_{L(G)}(g)] \rangle$

$$= \left\langle \left[2\sum_{r\neq j} \iota^+_{r_h} \wedge \iota^+_{j_h}, 2\sum_{r\neq j} m^-_{r_h} \vee m^-_{j_h} \right] \right\rangle$$

$$= \left\langle \left[2\sum_{r\neq j} \rho^+_h, 2\sum_{r\neq j} \rho^-_h \right] \right\rangle$$

$= \langle [2(k-1)\rho^+_h, 2(k-1)\rho^-_h] \rangle$.

So, $L(G)$ is $\langle [2(k-1)\rho^+_h, 2(k-1)\rho^-_h] \rangle$-regular.

Remark 4.5: According to graph theory in crisp sense, a regular graph's complement is also regular. However, a regular complement m-BPFG's need not also be regular. The example given below demonstrates this. In relation to the m-BPFG G in Figure 7., here

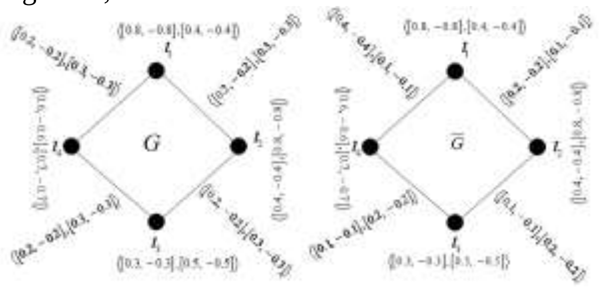

Figure 7: *G is a $\langle [0.4, -0.4], [0.6, -0.6] \rangle$-Regular but its complement $\bar{G}$ is not Regular*

$$Q = \left\{ \frac{\iota_1}{\langle [0.8, -0.8], [0.4, -0.4] \rangle}, \frac{\iota_2}{\langle [0.4, -0.4], [0.8, -0.8] \rangle}, \frac{\iota_3}{\langle [0.3, -0.3], [0.5, -0.5] \rangle}, \frac{\iota_4}{\langle [0.6, -0.6], [0.7, -0.7] \rangle} \right\},$$

$$R = \left\{ \frac{\iota_1\iota_2}{\langle [0.2, -0.2], [0.3, -0.3] \rangle}, \frac{\iota_1\iota_4}{\langle [0.2, -0.2], [0.3, -0.3] \rangle}, \frac{\iota_2\iota_3}{\langle [0.2, -0.2], [0.3, -0.3] \rangle}, \frac{\iota_3\iota_4}{\langle [0.2, -0.2], [0.3, -0.3] \rangle} \right\}$$

Thus $d_G(\iota_1) = d_G(\iota_2) = d_G(\iota_3) = d_G(\iota_4) = \langle[0.4,-0.4],[0.6,-0.6]\rangle$.
So, G is $\langle[0.4,-0.4],[0.6,-0.6]\rangle$- regular. But, in its complement $\bar{G}$, $\bar{Q} = Q$

$$\bar{R} = \begin{Bmatrix} \frac{\iota_1\iota_2}{\langle[0.2,-0.2],[0.1,-0.1]\rangle}, \frac{\iota_1\iota_4}{\langle[0.4,-0.4],[0.1,-0.1]\rangle}, \\ \frac{\iota_2\iota_3}{\langle[0.1,-0.1],[0.2,-0.2]\rangle}, \frac{\iota_3\iota_4}{\langle[0.1,-0.1],[0.2,-0.2]\rangle} \end{Bmatrix}$$

Then $d_G(\iota_1) = \langle[0.6,-0.6],[0.2,-0.2]\rangle$, $d_G(\iota_2) = \langle[0.3,-0.3],[0.3,-0.3]\rangle$, $d_G(\iota_3) = \langle[0.2,-0.2],[0.4,-0.4]\rangle$, $d_G(\iota_4) = \langle[0.5,-0.5],[0.3,-0.3]\rangle$. So $\bar{G}$ is not regular.
The complement of a regular m-BPFG is characterized as being regular by the below theorem.

Theorem 4.6: Let $G = (V,Q,R)$be an m-BPFG with $\langle[\rho_h^+,\rho_h^-]_{h=1}^m\rangle -$ regular of the k-regular crisp graph G^*. Iff Q is a constant function or if Q contains exactly 2 values so that precisely one node takes the highest positive relationship and the lowest negative relationship value, then $\bar{G} = (V,\bar{Q},\ \bar{R})$ is a regular m-BPFG.

Proof: Let $G = (V,Q,R)$ be an m-BPFG with $\langle[\rho_h^+,\rho_h^-]_{h=1}^m\rangle -$ regular of the k- crisp regular graph G^*. Again, let $\bar{G}$ be regular m-BPFG.

Case 1: There is nothing to show if Q is constant.

Case 2: Assume that Q is not constant. If possible let Q has more than two values.
Also let $P_h\text{o}\Psi_Q^+(\iota_1) < P_h\text{o}\Psi_Q^+(\iota_2) < P_h\text{o}\Psi_Q^+(\iota_3)$ and $P_h\text{o}\Psi_Q^-(\iota_1) > P_h\text{o}\Psi_Q^-(\iota_2) > P_h\text{o}\Psi_Q^-(\iota_3)$ for $\iota_1,\iota_2,\iota_3 \in V$.
Therefore, we may suppose without losing generality that,
$P_h\text{o}\Psi_Q^+(\iota_1) = \min\{P_h\text{o}\Psi_Q^+(\iota): \iota \in V\} = c_h^+$ and $P_h\text{o}\Psi_Q^-(\iota_1) = \max\{P_h\text{o}\Psi_Q^-(\iota): \iota \in V\} = c_h^-$.
Then $P_h\text{o}\Psi_Q^+(\iota_1) \wedge P_h\text{o}\Psi_Q^+(\tau) = P_h\text{o}\Psi_Q^+(\iota_1) = c_h^+$ and $P_h\text{o}\Psi_Q^-(\iota_1) \vee P_h\text{o}\Psi_Q^-(\tau) = P_h\text{o}\Psi_Q^-(\iota_1) = c_h^-$.
Therefore, $P_h\text{o}d_{\bar{G}}^+(\iota_1) = \sum_{\iota_1\neq\tau}[P_h\text{o}\Psi_Q^+(\iota_1) \wedge P_h\text{o}\Psi_Q^+(\tau) - P_h\text{o}\Psi_R^+(\iota_1\tau)]$

$$= \sum_{\iota_1\neq\tau} P_h\text{o}\Psi_Q^+(\iota_1) \wedge P_h\text{o}\Psi_Q^+(\tau) - \sum_{\iota_1\neq\tau} P_h\text{o}\Psi_R^+(\iota_1\tau)$$
$$= \sum_{\iota_1\neq\tau} c_h^+ - P_h\text{o}d_G^+(\iota_1)$$
$$= (k-1)c_h^+ - \rho_h^+$$

and

$$P_h\text{o}d_{\bar{G}}^-(\iota_1) = \sum_{\iota_1\neq\tau}[P_h\text{o}\Psi_Q^-(\iota_1) \vee P_h\text{o}\Psi_Q^-(\tau) - P_h\text{o}\Psi_R^-(\iota_1\tau)]$$
$$= \sum_{\iota_1\neq\tau} P_h\text{o}\Psi_Q^-(\iota_1) \vee P_h\text{o}\Psi_Q^-(\tau) - \sum_{\iota_1\neq\tau} P_h\text{o}\Psi_R^-(\iota_1\tau)$$
$$= \sum_{\iota_1\neq\tau} c_h^- - P_h\text{o}d_G^-(\iota_1)$$
$$= (k-1)c_h^- - \rho_h^-.$$

Again, $P_h\text{o}\Psi_Q^+(\iota_2) \wedge P_h\text{o}\Psi_Q^+(\iota_3) = P_h\text{o}\Psi_Q^+(\iota_2) > c_h^+$ and $P_h\text{o}\Psi_Q^+(\iota_2) \wedge P_h\text{o}\Psi_Q^+(\tau) \geq c_h^+$ for all $\tau \neq \iota_3$ in V.
So, $P_h\text{o}d_{\bar{G}}^+(\iota_2) = \sum_{\iota_2\neq\tau}[P_h\text{o}\Psi_Q^+(\iota_2) \wedge P_h\text{o}\Psi_Q^+(\tau) - P_h\text{o}\Psi_R^+(\iota_2\tau)]$

$$= \sum_{\iota_2\neq\tau} P_h\text{o}\Psi_Q^+(\iota_2) - \sum_{\iota_2\neq\tau} P_h\text{o}\Psi_R^+(\iota_2\tau)$$
$$> \sum_{\iota_2\neq\tau} c_h^+ - d_G^+(\iota_2)$$
$$= (k-1)c_h^+ - \rho_h^+.$$

Also, $P_h\text{o}\Psi_Q^-(\iota_2) \vee P_h\text{o}\Psi_Q^-(\iota_3) = P_h\text{o}\Psi_Q^-(\iota_2) < c_h^-$ and $P_h\text{o}\Psi_Q^-(\iota_2) \wedge P_h\text{o}\Psi_Q^-(\tau) \leq c_h^-$ for all $\tau \neq \iota_3$ in V.
So, $P_h\text{o}d_{\bar{G}}^-(\iota_2) = \sum_{\iota_2\neq\tau}[P_h\text{o}\Psi_Q^-(\iota_2) \vee P_h\text{o}\Psi_Q^-(\tau) - P_h\text{o}\Psi_R^-(\iota_2\tau)]$

$$= \sum_{\iota_2\neq\tau} P_h\text{o}\Psi_Q^-(\iota_2) \vee P_h\text{o}\Psi_Q^-(\tau) - \sum_{\iota_2\neq\tau} P_h\text{o}\Psi_R^-(\iota_2\tau)$$
$$< \sum_{\iota_2\neq\tau} c_h^- - d_G^-(\iota_2)$$
$$= (k-1)c_h^- - \rho_h^-.$$

So $d_{\bar{G}}(\iota_1) = \langle[P_h\text{o}d_{\bar{G}}^+(\iota_1), P_h\text{o}\ d_{\bar{G}}^-(\iota_1)]_{h=1}^m\rangle \neq \langle[P_h\text{o}d_{\bar{G}}^+(\iota_2), P_h\text{o}d_{\bar{G}}^-(\iota_2)]_{h=1}^m\rangle = d_{\bar{G}}(\iota_2)$.
That is in conflict with the notion that $\bar{G}$ is regular. Hence Q contains exactly 2 values.
Subsequently, suppose that several nodes obtain the greatest positive and lowest negative relation values. Let ι_2 and $\iota_3$be two such nodes.
Let $P_h\text{o}\Psi_Q^+(\iota_1) = \min\{P_h\text{o}\Psi_Q^+(\iota): \iota \in V\} = d_h^+$ and $P_h\text{o}\Psi_Q^-(\iota_1) = \max\{P_h\text{o}\Psi_Q^-(\iota): \iota \in V\} = d_h^-$.
Then $P_h\text{o}\Psi_Q^+(\iota_1) < P_h\text{o}\Psi_Q^+(\iota_2) = P_h\text{o}\Psi_Q^+(\iota_3)$ and $P_h\text{o}\Psi_Q^-(\iota_1) > P_h\text{o}\Psi_Q^-(\iota_2) = P_h\text{o}\Psi_Q^-(\iota_3)$.
Therefore, $P_h\text{o}\Psi_Q^+(\iota_1) \wedge P_h\text{o}\Psi_Q^+(\tau) = P_h\text{o}\Psi_Q^+(\iota_1) = d_h^+$ and $P_h\text{o}\Psi_Q^-(\iota_1) \vee P_h\text{o}\Psi_Q^-(\tau) = P_h\text{o}\Psi_Q^-(\iota_1) = d_h^-$ for all $\tau \in V$,
$P_h\text{o}\Psi_Q^+(\iota_2) \wedge P_h\text{o}\Psi_Q^+(\iota_3) = P_h\text{o}\Psi_Q^+(\iota_2)$ (or $P_h\text{o}\Psi_Q^+(\iota_3) > d_h^+$) and $P_h\text{o}\Psi_Q^-(\iota_2) \vee P_h\text{o}\Psi_Q^-(\iota_3) = P_h\text{o}\Psi_Q^-(\iota_2)$ (or $P_h\text{o}\Psi_Q^-(\iota_3) < d_h^-$, and $P_h\text{o}\Psi_Q^+(\iota_2) \wedge P_h\text{o}\Psi_Q^+(\tau) \geq d_h^+$ and $P_h\text{o}\Psi_Q^-(\iota_2) \vee P_h\text{o}\Psi_Q^-(\tau) \leq d_h^-$ for all $\tau \neq \iota_3$ in V.
By going on as described above, we know how to prove that $d_{\bar{G}}^+(\iota_1) \neq d_{\bar{G}}^+(\iota_2)$ which is a inconsistency to our supposition that $\bar{G}$ is regular.
So truly one node takes the greatest positive and the lowest negative relationship values.
Let Q, on the other hand, satisfy the theorem's hypotheses.

If Q is constant, let $Q(\iota) = \langle[c_h^+ , c_h^-]_{h=1}^m\rangle$ for all $\iota \in V$ where $c_h^+ \in [0,1]$ and $c_h^- \in [-1,0]$.

Otherwise, let $c_h^+ = \min\{P_h o\Psi_Q^+(\iota): \iota \in V\}$ and $c_h^- = \max\{P_h o\Psi_Q^-(\iota): \iota \in V\}$. Thus as Q contains exactly 2 values so that one node takes highest positive and lowest negative relationship values, we get for any $\tau \in V$,

$P_h o\Psi_Q^+(\iota) \wedge P_h o\Psi_Q^+(\tau) = c_h^+$ and $P_h o\Psi_Q^-(\iota) \vee P_h o\Psi_Q^-(\tau) = c_h^-$ for all $\iota \neq \tau \in V$.

Therefore, $P_h o d_{\bar{G}}^+(\iota) = \sum_{\iota\neq\tau}[P_h o\Psi_Q^+(\iota) \wedge P_h o\Psi_Q^+(\tau) - P_h o\Psi_R^+(\iota\tau)]$

$$= \sum_{\iota\neq\tau}[P_h o\Psi_Q^+(\iota) \wedge P_h o\Psi_Q^+(\tau) - \sum_{\iota\neq\tau} P_h o\Psi_R^+(\iota\tau)]$$

$$> \sum_{\iota\neq\tau} c_h^+ - P_h o d_G^+(\tau)$$

$= (k-1)c_h^+ - \rho_h^+$ and

$$P_h o d_{\bar{G}}^-(\iota) = \sum_{\iota\neq\tau}[P_h o\Psi_Q^-(\iota) \vee P_h o\Psi_Q^-(\tau) - P_h o\Psi_R^-(\iota\tau)]$$

$$= \sum_{\iota\neq\tau}[P_h o\Psi_Q^-(\iota) \vee P_h o\Psi_Q^-(\tau) - \sum_{\iota\neq\tau} P_h o\Psi_R^-(\iota\tau)]$$

$$> \sum_{\iota\neq\tau} c_h^- - P_h o d_G^-(\tau)$$

$= (k-1)c_h^- - \rho_h^-$.

This is obvious for all $\tau \in V$, i.e. $d_{\bar{G}}(\iota) = \langle[(k-1)c_h^+ - \rho_h^+, (k-1)c_h^- - \rho_h^-]_{h=1}^m\rangle$ for every $\tau \in V$. So, $\bar{G}$ is a regular m-BPFG.

Remark 4.7: First fundamental sequence may be used to rephrase the aforementioned theorem: Let $G = (V, Q, R)$ be an m-BPFG with $\langle[\rho_h^+, \rho_h^-]_{h=1}^m\rangle -$ regular of the k-regular crisp graph G^* with $f\xi_{bs}(G) = \langle\left[\left[s_{1_h}^+, s_{2_h}^+, \cdots, s_{k_h}^+\right], \left[t_{1_h}^-, t_{2_h}^-, \cdots, t_{k_h}^-\right]\right]_{h=1}^m\rangle$. Then the complement $\bar{G} = (V, \bar{Q}, \bar{R})$ is a regular m-BPFG iff either Q is a constant function or Q contains exactly 2 values so that $s_{2_h}^+ = s_{3_h}^+ = \cdots = s_{k_h}^+, s_{j_h}^+ < s_{1_h}^+ \leq 1$ and $t_{2_h}^- = t_{3_h}^- = \cdots = t_{k_h}^-, -1 \leq t_{1_h}^- < t_{j_h}^-, j = 1, 2, \cdots, k, h = 1, 2, \cdots, m$..

The following theorem characterizes the regularity of an m-BPFG of a cycle.

Theorem 4.8: Let $G = (V, Q, R)$ be an m-BPFG such that G^* is cycle. Thus G is a regular m-BPFG iff every node contains the equal adjacency sequence.

Proof: Assume that each node of G contains the equal adjacency sequence $\langle\left[\left[s_{1_h}^+, s_{2_h}^+, \cdots, s_{k_h}^+\right], \left[t_{1_h}^-, t_{2_h}^-, \cdots, t_{k_h}^-\right]\right]_{h=1}^m\rangle$. Thus, according to the adjacency sequence definition of G is $\langle\left[\left[s_{1_h}^+ + s_{2_h}^+ + \cdots + s_{k_h}^+\right], \left[t_{1_h}^- + t_{2_h}^- + \cdots + t_{k_h}^-\right]\right]_{h=1}^m\rangle -$ regular m-BPFG.

Conversely, let G be $\langle[\rho_h^+, \rho_h^-]_{h=1}^m\rangle -$ regular m-BPFG. Let the cycle G^* be $\tau_0\tau_1\tau_2\cdots\tau_n$.

Let $Q(\tau_0\tau_1) = \langle[a_h^+, a_h^-]_{h=1}^m\rangle$.

Then $d_G(\tau_1) = \langle[\rho_h^+, \rho_h^-]_{h=1}^m\rangle$ implies $Q(\tau_0\tau_1) + Q(\tau_1\tau_2) = \langle[\rho_h^+, \rho_h^-]_{h=1}^m\rangle$

i.e., $\langle[a_h^+, a_h^-]_{h=1}^m\rangle + Q(\tau_1\tau_2) = \langle[\rho_h^+, \rho_h^-]_{h=1}^m\rangle$.

i.e., $Q(\tau_1\tau_2) = \langle[\rho_h^+ - a_h^+, \rho_h^- - a_h^-]_{h=1}^m\rangle$.

Similarly, $Q(\tau_2\tau_3) = \langle[\rho_h^+, \rho_h^-]_{h=1}^m\rangle - \langle[\rho_h^+ - a_h^+, \rho_h^- - a_h^-]_{h=1}^m\rangle = \langle[a_h^+, a_h^-]_{h=1}^m\rangle$,

$Q(\tau_3\tau_4) = \langle[\rho_h^+ - a_h^+, \rho_h^- - a_h^-]_{h=1}^m\rangle$ and so on.

If G^* is an even cycle, then k is odd. So $Q(\tau_k\tau_0) = \langle[\rho_h^+ - a_h^+, \rho_h^- - a_h^-]_{h=1}^m\rangle$. This proves that the alternative edges have the equal relationship values. So every node has the equal adjacency sequence whose elements are $\langle[a_h^+, a_h^-]_{h=1}^m\rangle$ and $\langle[\rho_h^+ - a_h^+, \rho_h^- - a_h^-]_{h=1}^m\rangle$ in increasing order.

If G^* is a odd cycle, then k is even. So $Q(\tau_k\tau_0) = \langle[a_h^+, a_h^-]_{h=1}^m\rangle$.

Now, $d_G(\tau_0) = \langle[\rho_h^+, \rho_h^-]_{h=1}^m\rangle$ implies $Q(\tau_0\tau_1) + Q(\tau_k\tau_0) = \langle[\rho_h^+, \rho_h^-]_{h=1}^m\rangle$

i.e., $\langle[a_h^+, a_h^-]_{h=1}^m\rangle + \langle[a_h^+, a_h^-]_{h=1}^m\rangle = \langle[\rho_h^+, \rho_h^-]_{h=1}^m\rangle$

i.e., $[a_h^+, a_h^-]_{h=1}^m = \langle\left[\frac{\rho_h^+}{2}, \frac{\rho_h^-}{2}\right]_{h=1}^m\rangle$.

Therefore, $\langle[\rho_h^+ - a_h^+, \rho_h^- - a_h^-]_{h=1}^m\rangle = \langle\left[\frac{\rho_h^+}{2}, \frac{\rho_h^-}{2}\right]_{h=1}^m\rangle$.

So every node has the equal adjacency sequence.

5. Conclusions

Numerous issues in disciplines as diverse as electrical networks, shipping, specialist system, picture capture, and complex security have been effectively solved using graph theory. Several expansions of graph theoretic notions have been developed in recent years to explain the imprecision and uncertainty in network situations. A tool called an m-BPFG is envisioned as having a considerably higher capacity to cope with the inherent fuzziness of human knowledge with greater flexibility and accuracy. By incorporating adjacency sequences, the first and second basic sequences in an m-BPFG, our study has expanded the graph theoretical conclusions under a m-BPF environment. The idea of adjacency sequences is then used to construct an iff condition for an m-BPFG with at most 4 vertices to be regular. Additionally, there have been certain definitions of what it means for a regular m-BPFG's complement, line graph, and other properties to be regular. Some characterizations for the edge, totally edge, partially edge irregularity, etc. for an m-BPFG can be considered for further research. These findings may also be applied to other types of fuzzy graphs, such as intuitionistic fuzzy graphs, picture fuzzy graphs etc.

Author Statements:

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